

Can Abenomics revive Japan?

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Abstract

In 2012 the Japanese economy was in a very delicate situation. The global financial crisis of 2008 followed by the Eurozone crisis since 2010, the disaster of 2011, the slowdown of the Chinese economy in 2012 were keeping the yen high and exports and enterprises investment were very weak. Japan was in a deflation spiral with falling profits and wages. Only stimulus policies to consumption and housing started in 2008 were keeping the economy afloat, but the government could not afford to continue increasing its huge debt to pay for them. An ageing and falling population was not helping either. By the summer of 2012 it become clear that drastic changes were needed, something had to be done.

Promising to revive Japan with a huge monetary stimulus and big public works the Liberal Democratic Party returns to power and the new Prime Minister Shinzo Abe started implementing these policies immediately. Structural reforms would join later. These policies have been labeled “Abenomics” and there is great expectation about their ability to revive Japan from both domestic and international economic players.

This paper analyzes the effects of the monetary and fiscal stimulus of the “Abenomics” and we show that these policies could succeed by making the economy grow faster and by turning inflation into positive territory again. Later in time, the government would achieve a surplus and the debt to GDP ratio would start falling. However, if the proper structural reforms do not take into effect soon enough, the economy would overheat, the monetary stimulus would have to be reversed causing instability and growth would fall below its potential once again.

We also study the impact of the consumption tax hike and we see that it can help reduce the deficit more quickly but growth slows down. For this reason, the existent safety net to support low income workers should be preserved and probably expanded.

Keywords: Abenomics, quantitative easing, deflation, deficit, debt, demographics

Introduction

Between 2002~2007 the Japanese economy managed to have almost uninterrupted expansion with an annual average growth rate of 2%. The Japanese authorities were confident enough to start dealing with its debt and deflation problems. The future looked promising again. However, all hopes came to an end in 2008 when the burst of the housing bubble in the United States was sending shock waves across the world and the crisis became global. In the fiscal year 2008 Japan's exports fell 37% and the GDP contracted 9%. Fearing the world economy could fall into a Great Depression like the one experience in the 1930s, world leaders responded with a rare coordinated monetary and fiscal stimulus and the year 2009 was the beginning of the world recovery. Japan also implemented expansionary policies. The economy improved that year. However, by 2010 the European crisis was causing the yen to rise what was hurting exports again. While this was ongoing, on Friday March 11, 2011 a magnitude-9 earthquake stroke Japan in the Tohoku region triggering a tsunami and causing a nuclear accident in the Fukushima plant. The disaster killed almost nineteen thousand people, destroyed public and private infrastructure and left the country without nuclear power. One year later, the slowdown in the Chinese economy and territorial disputes with that country caused exports from Japan to China fall about 25% in one year.

By the middle of 2012 it became clear that drastic changes were needed. In December the Liberal Democratic Party of Japan (LDP) won the lower house elections, Shinzo Abe became the Prime Minister and start putting in place three arrows of the Abenomics. The first arrow is a huge monetary stimulus. The second arrow is a fiscal stimulus by expanding public works. The third arrow are the structural reforms which include deregulation, flexibilization in the labor market, stimulating female labor, joining the Transpacific Partnership negotiations were also announced.

The government also started pushing for a restart of those nuclear reactors complying with the new and stricter safety standards. The government was also showing confidence in its stimulus plan by announcing the fiscal situation was to be repaired by going ahead with the previously agreed consumption tax hike expected in 2014 and 2015.

In the first six month the new government has been able to change expectations. The stock market has soared and the value of the yen has fallen. Consumption and housing investment are in an upward trend and exports are slowly responding to the lower value of the yen. As we show later, the recovery in domestic demand should be seen as a continuation of the trend observed since 2009 when the government started a series of

stimulus policies, rather than an exclusive consequence to the Abenomics.

The objective of this paper is to study how the monetary and fiscal stimulus could affect the future growth path of the Japanese economy. We attempt to answer some of the following questions. Can the monetary and fiscal policy achieve higher growth and healthy inflation? Can these policies reduce the debt burden? How does an increase in the consumption tax affect the performance of the economy?

To answer these questions we use a similar model employed in a previous work (Bardas, 2011) which considers a dynamic macroeconomic model for the behavior of the economic agents and their interaction through the markets. We used the estimations of the parameters of that same paper and made simulations to observe the economic path for twenty years. Our model considers the demographic structure Japan's population to reflect the effects of the falling and ageing population on economic growth and public finances.

As we show, although the expansionary monetary and fiscal policies are appropriate for the prevailing conditions, the scale of the monetary policy can be destabilizing unless the proper structural reforms increase the potential growth rate of the economy. Details of the third arrow have not been completely disclosed yet, and we do not analyze it directly. However without the proper structural reforms, after a period of higher growth, the economy will slowdown and unemployment rate will start rising again.

The paper is organized as follows. In section 1 we describe the economic conditions prevailing in 2012 and the Abenomics three arrows. In section 2 we describe the our theoretical model. In section 3 we present the hypothesis and results of our simulations. Last we obtain some conclusions and describe the limitations of our work.

1. Is Japan rising again?

The Abenomics is a drastic economic policy which is trying to change the course of the Japanese economy under unfavorable international scenario and many internal constraints. External conditions are characterized by a world economy still trying to emerge from the 2008 global financial crisis. Global growth rate is at a low 3% per year. Uncertainty about the final resolution of the global crisis has made investors send their funds to Japan causing a rise in the yen making even more difficult to export from Japan. Among the most visible internal constraints are the high public debt and the ageing and falling population.

1.1. The last four shocks to the Japanese economy

Since 2008 the Japanese economy suffered four important shocks which left it in a very fragile situation. The first shock was the global financial crisis of 2008, which became visible late in the summer of 2007 with the collapse of the housing bubble in the United States. The USA crisis sent wave shocks around the globe leaving the economy of the developed countries in a weak condition which still persists in the present. The global economic retreat meant lower demand for Japanese products and uncertainty about the how the crisis would be resolved lead to a flow of capital towards Japan causing a 25% rise in the JPY. Falling income in the rest of the world and a rising yen caused a more than 35% fall in Japan's exports in just one year. Fearing the world could fall into a depression like in the 1930s, world leaders cooperated by unleashing expansionary fiscal and monetary stimulus. The stimulus policies worked and in 2009 the world economy was improving and Japan's exports were growing again

The recovery was not completed yet when by the end of 2009 it became evident that the global crisis had been spreading into several European countries like Iceland, Ireland and Greece, later to Portugal, Spain, Italy, and Cyprus. Because of the delicate fiscal position of those countries, the European Union adopted austerity measures as a signal to investors that finances were going to be corrected. However, these austerity measures were not balanced with the appropriate growth policies and the block's economy has since then remained very weak. This became a second "shock" to the Japanese economy, as uncertainty among investors produced continued flows of capital to Japan the yen rose further, exports slowed down and profits of Japanese multinationals obtained at their overseas factories valued less in yens when repatriated.

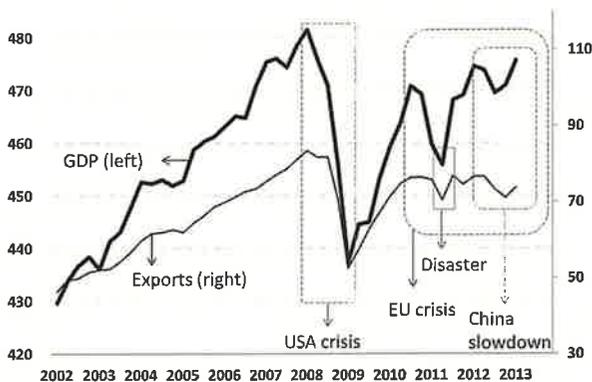
The third shock came from inside Japan in March 11, 2011 when an earthquake followed by a tsunami killed more than 19,000 people, destroyed important infrastructure in the Northeast Japan, and caused a nuclear plant accident. Supply chain disruptions caused by the disaster had a negative impact in the volume of exports once more. Because of safety concerns about the rest of the nuclear plants, the government ordered the shutdown of all of them, leaving Japan without nuclear power. Later only two plants were allowed to reopen.

The fourth shock comes from China in 2012. As the Chinese authorities try to rebalance its economy shifting growth from investment and export to consumption, and enforces higher environmental standard, growth rates have fallen to 7% from 10%. Territorial disputes between Japan and China which caused a boycott of Japanese products added to the problem.

As shown in Figure 1 real exports have not completely recovered since the global financial crisis, in the first quarter of 2013 are still 12% below its peak of the first

quarter of 2008. In spite of all these negative shocks, supported by several stimulus monetary and fiscal measures by the Japanese authorities, domestic demand managed to achieve a better performance than external demand but the GDP has not returned to its peak of 2008 yet.

*Figure 1: The last four shocks to the Japanese economy
GDP and exports (real values in trillion yens, at 2013-Q1 prices)*



Source: National Accounts Cabinet Office, Government of Japan

1.2. The Abenomics three arrows

During the 2012 election campaign for the lower house, the Liberal Democratic of Japan (LDP) promised very drastic changes in the economic policy and in December was able to return to power. The policies of the new Prime Minister Shinzo Abe, the “Abenomics” include three arrows. The first is a big expansion of the amount of money through quantitative easing to reflate the economy. Haruhito Kuroda was nominated as President of the Bank of Japan and immediately started an unprecedented monetary stimulus which is supposed to double monetary base in two years and achieve an annual inflation rate of 2%. The second arrow is a ¥10 trillion expansion in public infrastructure. The third arrow are structural reforms, which include deregulation, flexibilization of the labor market, stimulus to female to join the labor force, improving agricultural efficiency. As part of these measures, Japan decided to join the Trans Pacific Partnership (TPP) negotiations. Though very controversial among farmers, the government was somehow able to convince the public that some agricultural products will be protected and in July, 2013 joined the 18th round of negotiations in Malaysia.

Japan has one of the highest ratios of government debt to GDP in the world and to

calm investors, the new government has suggested it will go ahead with the planned increase in the consumption tax in 2014 from 5% to 8%, and later in 2015 to 10%, though it has also said that timing could be changed depending on economic conditions.

1.3. Expectations of the public are changing: rising stocks and falling JPY

The new authorities have made a good job in changing the public expectations at least if measured by the stock prices and the exchange rate (Figure 2). Investors have been buying stocks and foreign currencies. According to the plan, as the monetary stimulus keeps interest rates low, public works will revive the economy and inflation would return. As investors believe the interest rate will not rise, the JPY will fall once the world economy improves, and interest rates in other big economies like the United States start rising again. It is in the anticipation of a falling JPY that investors have been buying foreign currency, what in turn has caused a fall of about 25% in six months. At the same time, with the expectation of higher dividends, investors' purchases of stocks made the Nikkei index rise 75% at its peak. As the BOJ continues buying assets in the market, investors' funds are freed to be canalized to the stock and foreign exchange market. Purchases of bonds by the BOJ may have been insufficient to match the amount sold by private investors what has been causing a temporal rise in the yields and some volatility in the market. This made investors less cheerful and both the stock market and the foreign exchange market experienced a downward correction by the end of May.

Figure 2: Change in investor's expectations
Nikkei stock index and exchange rate (1 USD: JPY)

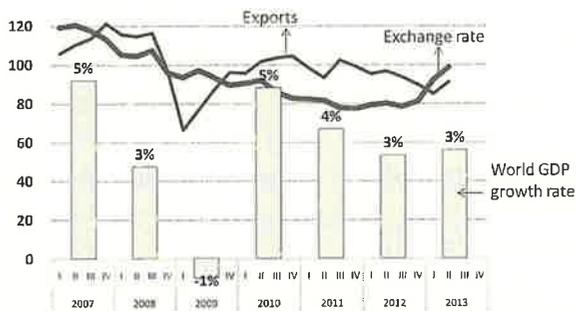


Source: Nikkei stock index from Nikkei Shinbun. Exchange rate from BOJ.

1.4. Sluggish foreign demand and yen depreciation

From Figure 3 it is clear that the slowdown and fall in the world GDP in 2008~2009, jointly with the appreciation of the yen have had a negative impact in Japan's exports. As the world coordinated their monetary and fiscal policies the global economy recovers and Japan's exports grow again in 2009. Because of the European crisis since 2010 and the Chinese economy slowdown in 2012, exports remain very weak and the yen stayed high. Abenomics is trying to reverse this trend, but the sharp fall in the value of the yen during first semester of 2013 though has translated in an increase in the nominal value of exports in yens, it has not produced significant increase in the volumes of exports yet. If the global economy manages to keep its growth rate above 3%, the depreciation of the yen may start having a positive effect in the second semester.

Figure 3: Japan exports (volume index) may start responding to the falling yen but global growth remains weak



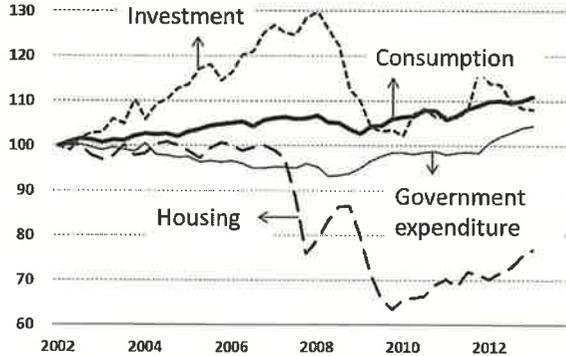
Source: exchange rate (\$1: ¥), BOJ, world GDP growth (Purchasing Parity Power, % rate), International Monetary Fund

1.5. Rising consumption and housing under poor labor conditions.

From Figure 4 it can be seen that domestic demand has been recovering since 2009. Consumption, housing and government expenditures are slowly expanding but this is not due to the Abenomics, but rather due to a series of stimulus policies started in Japan in 2009: monetary stimulus from the BOJ of the previous administration has been reducing the interest rate, the government had also started a program of subsidies to the purchases of low energy consumption electric appliances, subsidies and tax credits to environmental friendly cars, tax credits to purchases of a first home and free high-school among other measures.

Private investment in general has followed the export trend as companies wait for signs of a higher pace of global expansion before ordering new equipment. We show later that this means the existence of important idle capacity.

*Figure 4: Domestic Demand has been slowly improving since 2009
(real quarterly index, at 2002-Q1 prices)*

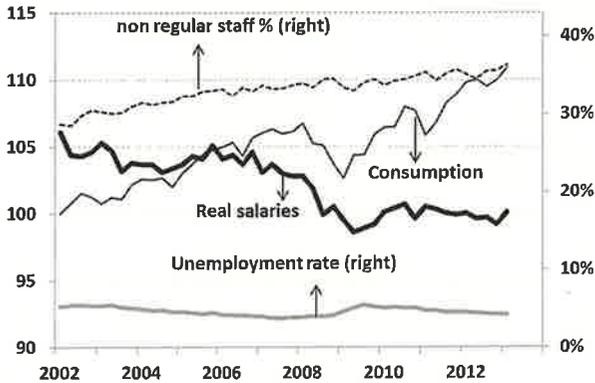


Source: National Accounts, Cabinet Office, Government of Japan

Figure 5 helps sustain the hypothesis that government stimulus policies are behind consumption and housing recovery. Although the unemployment rate has been falling very slowly since 2009, labor conditions have not improved. Since 2008 the real average salary has remained relatively subdued as proportion of non-regular staff continues rising. Some of the stimulus policies have already expired but free high-school education and tax credits to home and efficient cars still remain. More recently the rise in stock values in the first half of 2013 produced an increase in the value of the household's wealth and must have released the desire to expand spending. Also the imminent consumption tax hike may be making the household change their purchases plans on housing and durable goods, bring forward future purchases to avoid the tax increase. It is also possible that public inflation expectations have change and with rising prices, the household finds optimal to consume sooner than later.

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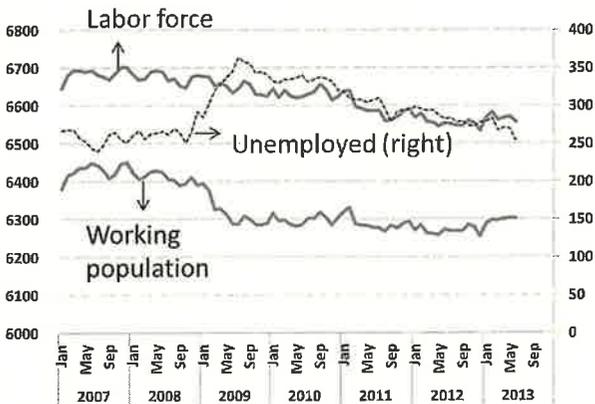
Figure 5: Labor conditions remain poor but consumption is growing



Source: e-Stat and Cabinet Office

The labor market picture becomes clearer if we take a look at Figure 6, what shows that the fall in the number of unemployed has come with a reduction in the labor force not with a visible increase in the working population. In June, 2013 both the labor force and the working population (which includes employed and self-employed) are 1.5 million people lower than in its peak in 2007.

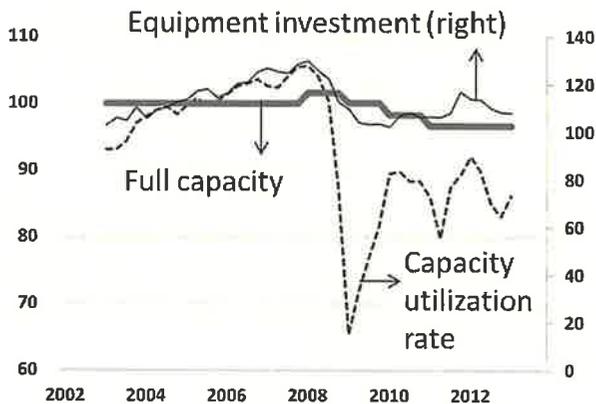
Figure 6: Labor force and working population (ten thousand people)



Source: e-Stat, labor force survey

In spite of the stimulus coming from very low interest rate and the prospects of higher inflation, private investment in machinery, equipment, factories, offices and other buildings is not growing yet. This does not necessarily mean that enterprises doubt Abenomics can work, but rather are a reflection of low capacity utilization. In the first quarter of 2013 there is still a 10% of idle capacity (Figure 7). If the recovery continues, enterprises will use more capital, and part of this equipment will depreciates by obsolescence.. Until then, investment will be lagging in its recovery.

Figure 7: The economy is operating below capacity and enterprises do not invest



Equipment investment (right axis): real index from National Accounts.
 Full capacity and capacity utilization rate are measured in the left axis.
 Source: Ministry of Economy Trade and Industry and Cabinet Office

1.6. The difficult task of reflating the economy

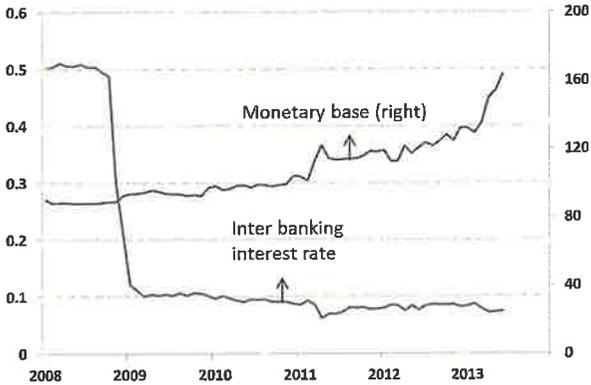
The most recent data shows that the inflation rate has turned positive. However it is too soon to say whether this can be count as an achievement for the government because most rising prices are due to rising costs of energy and other imported stuff, which has been caused by a falling yen. As long as there are no meaningful and continuous wage increases, it is doubtful that a healthy inflation lead by rising demand can be materialized.

The central bank has been fulfilling his promise and increased the monetary at a pace of ¥10 trillion per month. The inter-banking lending rate stayed almost unchanged below 0.1% and the average lending rate by city banks close to 0.6%~0.8%. We estimated the real interest rate assuming perfect foresight for the annual inflation rate

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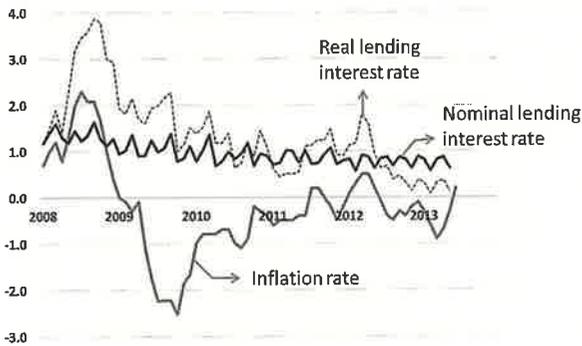
(for the period January~July 2013 the expected forward inflation rate is 0.2%~0.5%).
The increase in the inflation rate would result in lower real interest rates.

Figure 8: The BOJ increases the monetary base



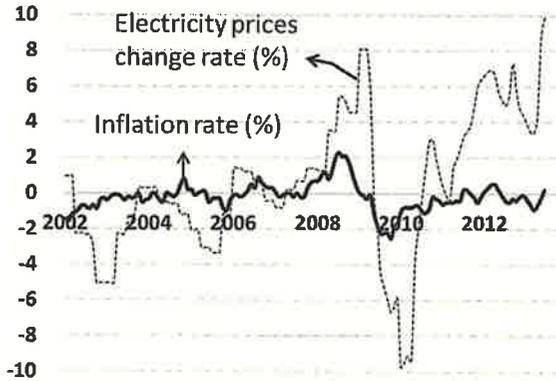
Monetary base (trillion JPY) and inter-banking interest rate (annual %)

Figure 9: The real interest rate has been falling



Source: Interest rates, BOJ. Inflation rate, Statistics Bureau

Figure 10: Price of energy and other imported stuff is rising fast

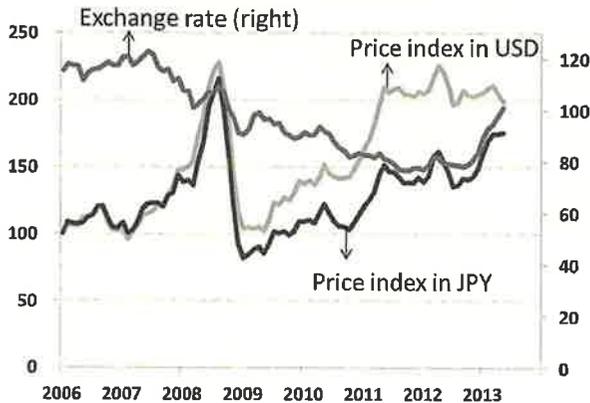


Source: Ministry of Internal Affairs and Communications (e-Stat)

The consumer price index (Figure 10) shows that the overall inflation rate though close to zero, is slightly positive. This is mainly due to the rising cost of energy and other imported stuff. Prices of the other goods and services produced in Japan are still falling.

It is interesting that the rise in energy prices is not explained by rising international prices. The average price index of imported energy measured in USD has been falling since the end of last year, then as Figure 11 shows the rise is due to the falling yen.

Figure 11: The falling JPY has been driving energy prices up.



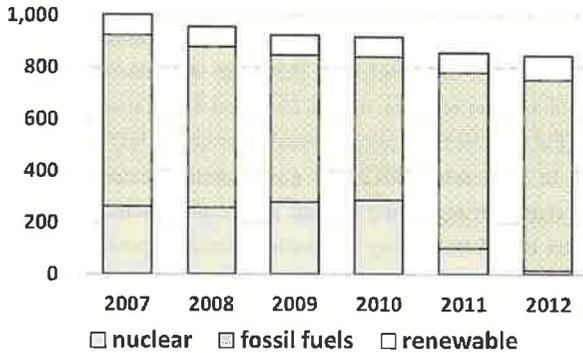
Source: BOJ

1.7. Japan's demand for fossil fuels is rising

Because of the Fukushima nuclear plant accident in the in March 11, 2011, the energy mix (fossil fuels, nuclear, renewable) changed drastically from (60%, 30%, 10%) to (88%, 2%,10%). The reactors of that plant were decommissioned and of the remaining total 50 nuclear reactors, only 2, Nr. 3 and 4 of Kansai Electric Power Co. in the town of Ōi, Fukui Prefecture were allowed to restart in July 2012 and are operating in the present. In September 2012, the government created a more independent organization in charge of regulating nuclear power, the Nuclear Regulation Authority (NRA). The NRA issued new safety standards for nuclear power, which were put into force in July 2013. Now, the government is pushing for restarting operations of those plants which can prove they comply with the new standards. However, the situation in the Fukushima nuclear plant has not been resolved yet, contaminated water is still leaking into the sea. The government decided to get involved and the International Atomic Energy Agency continues to show its readiness to help. We think the public may not be persuaded so easily about restarting the nuclear power.

Because Japan has almost no energy resources and it has to import most of the oil, coal and natural gas it consumes, the falling of the yen has made energy more expensive for companies and households. The increase in energy costs has been specially felt by non-export enterprises like services providers, what is worrisome as these enterprises represent an important part of the economy. As can be observed in Figure 12 because the economy remains weak, total consumption of electricity is much lower than before the 2008 global financial crisis. Because of the suspension of the production of electricity from nuclear power, the share of imports of fossil fuels on the total imports has increased from 28% in 2010 to 35% in the first semester of 2013. Once the economy starts recovering demand for energy will grow and, if the nuclear reactors are not restarted, Japan will have to increase purchases of energy sources in the international market.

Figure 12: Japan's total consumption of electricity (billion Kwh)



Source: Agency for Natural Resources and Energy of the METI

The development of shale gas in the United States has risen expectations of a relief for Japan too if it manages to negotiate a stable supply of these resources. Reaching an agreement is not guaranteed as the United States exports natural gas only to countries with which it has a FTA. Though there is a bill in the United States Senate to allow exports of liquefied natural gas to NATO allies and Japan even without a FTA, it has not been approved yet. If Japan joins the Trans Pacific Partnership this hurdle might be cleared.

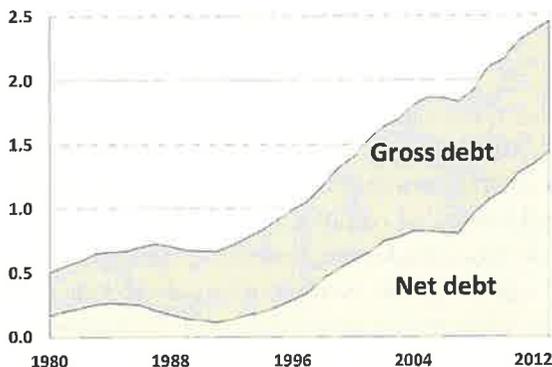
1.8. The debt of the government is high, very high

The overall deficit of the government is 10% of the GDP, much higher than the 3% in 2007 before the global financial crisis. The total tax collection (including social security contributions) has fallen 8% since then, but social security transfers rose about 13% due to the aging population and the increase in other subsidies and transfers to the households. On the top of this, the expansion in ¥10 trillion yens of public works will all add to the deficit in 2013.

The rise of the debt of the government can be tracked down to the 1990s when the economy was very weak and needed government support. Because of the global financial crisis of 2008, the debt to GDP ratio took another jump and as shown in Figure 13, the ratio of the gross debt of the general government to the GDP has reached 2.4, the highest in the world. The ratio of the net debt to GDP is 1.4, only second to Greece which stands at 1.7. The absolute value of the gross debt surpassed the ¥1 quadrillion just this week.

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Figure 13: The rise of the public debt



Source: IMF

Any increase in the nominal interest would accumulate to the debt and the situation could get out of control if investors doubt the government plan and start demanding a premium and the nominal interest rate rises. For the moment, the average nominal interest rate paid by the government bonds is at a historical low level of 1.2% (40 and 30 year bonds: 2.2%, 20 year bond: 2%, 10 year bond 1.4%, 5 year bond 0.5%). Most of the debt (90%) is owned by domestic agents: 62% by financial institutions, 11% by the BOJ, 3% by the enterprises pension fund, 5% by the households, and 10% by the public pension fund and other government agents (Figure 14). Although 54% of the debt expires in less than 5 years, the government can still borrow cheaply and renew its debt. Since April 2013 the BOJ has started a new round of quantitative easing (QE), and has already purchased about ¥25 trillion of government bonds in the secondary market, thus the BOJ owns close to 14% of the government debt.

Figure 14: Holding of the government debt by type of creditor (September 2012)

Other government agencies	3%
Public Pension Fund	7%
BOJ	11%
Private banks	42%
Insurance companies	19%
Enterprises pension fund	3%
Foreign investors	9%
Household	3%
Other	3%

Source: Ministry of Finance

1.9. Risks of the Abenomics policies

Some economists like Joseph Stiglitz and Paul Krugman have both praised the Abenomics and sound optimistic about its effectiveness. Other economist like Martin Feldstein say that the fall in the yen will push costs up and inflation could cause nominal interest rate to rise and the fiscal position would worsen.

There are some risks implicit in the Abenomics and risks that threaten the Japanese economy in general. The government has moved forward with the first and second arrow of the policies which are expansionary monetary and fiscal policies. The third arrow or structural reforms could have the potential of increasing the long term growth rate and the high expectations have been put on the announcement of its details and in its implementation. Within the structural reforms, one consistent move is that Japan has decided to join the TPP negotiations and participated in the meetings in Malaysia in July 2013. The government has said that it will try to keep some kind of protection to sensitive products like rice, wheat, sugar, dairy products, and beef, but is not clear the negotiating partners will accept, nor even if the United States consents about it. With respect to the other reforms, many announces have been made and several meeting have taken place, but real progress is still to be seen.

Investors have reacted more or less positively so far, but if no progress is shown in these structural reforms, investors could start doubting about the long term growth of the economy and even about the sustainability of the whole program. The results of the simulations we performed and that are summarized in section 3, also show that the monetary and fiscal policy are unable to keep the economy growing for longer than 8 or 10 years and structural reforms must be implemented, the sooner the better.

The first and second arrows of the Abenomics are supposed to work with the third, and without it they have some risks. The monetary stimulus intends to keep the nominal interest rate as low as possible, and by inducing inflation to reduce the real interest rate. This would stimulate investment and when the economy starts growing, the value of the debt would fall in relation to the size of the economy. However, there is still great uncertainty about whether the BOJ can succeed on keeping the yield of the government bonds low through the quantitative easing. If investors start having doubt about the program, they could start demanding a higher premium for holding government bonds (The Economist, 2013).

In any case the monetary policy will not be enough because demographic factors are making the fiscal position unsustainable even if the economy picks up. As the number of retirees rises and the number of workers fall, the deficit of the social security will continue rising. This will mean that the debt will may not fall as a proportion of the GDP and the authorities will have to convince investors that it is serious about

repairing the delicate fiscal position. In this regard, the government still has to make the final decision about the consumption tax hikes which are expected in April of 2014 and October of 2015, but postponing the rise will cast doubt about the whole program, and by scaring investors the economy could be derailed. The massive monetary stimulus brings more risks because the resultant falling yen means higher costs of energy and raw materials. This would pose no threat if exports and profits start growing again, but for the moment, signs of consistent growth are mixed and elusive. Furthermore, although a lower yen makes more profitable the proceeds of overseas investment when they are repatriated, by inducing investors to send their funds to the rest of the world, higher volumes of assets held abroad also mean that in the event of any external negative shock higher volumes of funds can suddenly return causing unexpected rises in the value of the yen like in 2008.

There are also risks to the global economic and political situation which Japan has very small capacity of control. First, the United States economy has improved but it remains growing rather slowly, the Eurozone may come out of recession sooner or later but no rosy recovery is expected. The Chinese economy has been slowing down and risk of a banking crisis in this country is still present even if the Chinese authorities have more capacity of controlling their economy than other democratic countries. Territorial disputes with China may not be resolved soon, and this adds to the negative factors that keeps Japan exports weak. Since the shutdown of its nuclear plants, Japan's dependency on fossil fuels from the Middle East has increased. Instability in the region is not diminishing and spikes in the price of oil could happen at any time.

2. A Model to Analyze the Long Term Prospects for the Economy

In this section we present a summary of the model we use to analyze the effects of the Abenomics in the long term performance of the economy. We refer to Bardas (2011) for the details of the model, its formal mathematical representation and the estimation of the parameters and exogenous variables. Some parameters have been recalibrated to make the year 2012 as the benchmark, instead of 2011 as in Bardas (2011).

The model includes seven economic agents and four markets. Economic agents are a representative working household, a representative retired household, the enterprises, the labor unions, the government, the central bank and the rest of the world. The households decide how much to consume, save and work, and how to allocate their assets. The enterprises decides how much to produce, how many workers to hire and how much to invest. Labor unions negotiate with the enterprises the nominal wage

rate. The government provides public services, administers the social security system and is in charge of the fiscal policy. The central bank, the Bank of Japan decides on the monetary policy. The rest of the world is provider of imported stuff and is purchaser of Japanese products. It is also the destiny and/or origin of Japanese direct and financial investment.

Economic agents interact with each other through the market of goods and services, the money market, the foreign exchange market and the labor market. The model is variation of more traditional open economy dynamic Aggregate Demand and Aggregate Supply (AD-AS) models. By including negotiations between labor unions and enterprises, the labor market is not necessarily at equilibrium, the rest of the markets are. The main variables of the model include among others, the GDP, the price level, the inflation rate, the nominal and the real interest rate, the nominal and real wages rates, the level of employment, the unemployment rate, the exchange rate, consumption, private investment, exports, imports, the government expenditure, capital flows, taxes and the debt of the government.

In the following sections we describe the behavior of the economic agents and their interaction through the markets. A list of the variables, parameters and a summary of the formal mathematical representation of the model is presented in the Mathematical Appendices.

2.1. The households

As in Bardas (2011) section 2.2., the representative working household is assumed to maximize her life utility subject to the time and budget constraints. Utility or happiness is obtained from consumption and leisure. The time constraint indicates that the total time available is allocated between leisure and work. The life budget equalizes the total net present value of consumption to the total net assets plus the net present value of after tax income. Income is obtained from labor and transfers from the government (health, education, unemployment benefits, public assistance, etc.). The optimization problem may be represented as follows.

$$\begin{aligned} & \text{MAX Utility (consumption, leisure)} & (1) \\ & \text{Subject to: time constraint \& budget constraint} \end{aligned}$$

The retired household does not work and all the available time is assigned to leisure. Therefore, the households optimizes her life utility from consumption subject to her budget constraint: assets plus the net present value of all pensions and other health or public assistance transfers must equalize the net present value of consumption.

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The solution to the household optimization problems, give us the three functions below, the consumption function, the saving function and the labor supply function, depending on the value of assets, the present and expected future stream of real wages, real interest rate, transfers (pensions, health, public assistance, etc.) and taxes.

$$\text{consumption} = f(\text{assets, real wage, transfers, real interest rate, taxes}) \quad (2)$$

$$\text{saving} = f(\text{assets, real wage, transfers, real interest rate, taxes}) \quad (3)$$

$$\text{labor supply} = f(\text{assets, real wage, transfers, real interest rate, taxes}) \quad (4)$$

2.2. Enterprises

Enterprises employ workers and capital to produce goods and services in order to obtain maximum profits. When solving their optimization problem, the enterprises faces two technological constraints, the production function and the requirement of imported materials. The requirement of raw materials and energy are related to production by a fixed coefficient technology. As in Bardas (2011) section 2.4, the optimization problem can be represented as follows.

$$\text{MAX Profits} \quad (5)$$

Subject to: Production function & requirement of imported materials

The production function relates total production to labor and capital which includes private capital and public infrastructure as well as land. Though private capital is an input to the level of production, companies are not assumed to adjust it to its optimal level. Instead, each year, the level of capital applied in the production process is the capital stock accumulated by the end of the previous period. Enterprises may not be able to use an optimal level of capital, so in some periods capital utilization can be below capacity. Enterprises can purchase new equipment and machinery by considering alternative investment projects. The gross level of private investment is determined as the net present value of future expected profits to be obtained from those projects. Therefore, the level of investment will be a function of the expected profits, the expected real interest rate, the risk rate and the expected growth rate of profits.

The solution to the profit maximization problem gives the labor demand, the supply of goods and services represented by the GDP, all functions of the wage rate, the price level, the exchange rate, and the price of imported materials. Private investment depends on the expected real interest rate, growth, risk and the available capacity.

$$\text{labor demand} = f(\text{real wage rate, exchange rate}) \quad (6)$$

$$\text{GDP} = f(\text{real wage, exchange rate}) \quad (7)$$

$$\text{imported energy and raw materials} = f(\text{GDP}) \quad (8)$$

$$\text{investment} = f(\text{expected real interest rate, growth, risk}) \quad (9)$$

2.3. Relations with the Rest of the World

Trade relations with the rest of the world are described by the import and export functions like in Bardas (2011) section 2.7. Investment relations are described by the financial flow of capital and the foreign direct investment functions (Bardas, 2011, section 5).

The import function

The import function represents the total volume of goods and services Japan buys from other countries. As explained before, imports of raw materials and energy are computed as a constant coefficient of the volume of production. Imports of final goods and services compete with local production and it is assumed that domestic agents will decide how much to buy of each by minimizing the cost of total purchases, subject to a certain preference function as described below.

$$\text{Min Cost} = \text{cost of local purchases} + \text{cost of imported stuff} \quad (10)$$

$$\text{Subject to: Preference} = f(\text{imported stuff, domestic stuff, raw materials})$$

The solution to the above optimization problem will give the demand of imported stuff as a function of Japan's GDP and the real exchange rate.

$$\text{imports} = f(\text{GDP, real exchange rate}) \quad (11)$$

The export function

The export function represents the demand for Japanese goods and services by the rest of the world. The volume of exports is an increasing function of the real GDP of the rest of the world and the real exchange rate. The real exchange rate is the ratio of international prices expressed in JPY to the GDP deflator which measures the price level of Japanese producers.

$$\text{exports} = f(\text{global GDP, real exchange rate}) \quad (12)$$

Financial flow of capital and foreign direct investment

Investors and enterprises diversify their portfolio allocating their financial and real

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capital both in Japan and in the rest of the world. When the international interest rate is rising relatively to the interest rate in Japan or also when the risk rate abroad falls respect to the risk rate in Japan, investors send their money abroad. The expectation of a lower value of the JPY also makes investors send their money abroad. Direct investment depends on the relative rate of return on physical assets, the risks rates and the change in the value of the currency. The flow of financial assets (FFA) and the foreign direct investment (FDI) function depend of the arbitrage rate as follows:

$$\text{FFA} = f(\text{arbitrage rate}) \quad (13)$$

$$\text{FDI} = f(\text{arbitrage rate}) \quad (14)$$

The arbitrage rate for the FFA is the addition of the interest rate differential the currency depreciation rate and the risk differential. The arbitrage rate for FDI considers the profit rate differential instead of the interest rate differential. The interest rate of the rest of the world is computed as the weighted average of the main destinies of Japan financial investment abroad.

2.4. The Government, the BOJ and Labor Unions

The government provides services, administers the public infrastructure, is in charge of the social security fund and implements the fiscal policy. Taxes and debt expansion are used to finance the total expenditures. Collected taxes are the consumption tax, the income tax, the inhabitant tax, the corporate tax, the asset tax, import tariffs and the social security contributions. Japan's social security expenses include pensions, health, public assistance, care for the elderly and unemployment benefits. Social security contributions are paid by workers and enterprises almost fifty-fifty.

The Bank of Japan is in charge of the monetary policy. It controls the monetary base in order to achieve a certain target inflation rate while also trying to keep the economy growing and the unemployment rate low. The monetary policy is based on the open market operations by the BOJ at the inter-banking lending market which tries to keep the overnight interest rate under a certain targeted range consistent with the inflation target. The current President of the BOJ has established a 2% of annual inflation rate. The monetary base can also be expanded by quantitative easing which consists in the purchases of public and private assets in the secondary market to increase the quantity of money, keep the interest rate low and stimulate borrowing.

Although both the BOJ and the government may intervene in the exchange market, it is only in very short periods of high volatility in the market, in general the market operates freely according to supply and demand.

Labor unions take into account the policy objectives of the BOJ and the fiscal policy by the government when trying to fight for better working condition for all workers. At each annual round of negotiations labor unions and the business federation negotiate the new level of the wage rate according to the inflation rate and productivity changes. Labor unions bargaining power is higher when unemployment is low and the annual adjustment of the wages depend on the past and expected inflation, the unemployment rate, productivity and enterprises profits.

2.5. The Markets, Dynamics and Expectations of the Agents

Four Markets

There are four markets in our model: the labor market, the market of goods and services, the money market and the foreign exchange market. The economy is described by a dynamic open economy Aggregate Demand – Aggregate Supply model which evolves according to the changes in population, capital accumulation, technological progress, global growth and economic policy.

Labor supply comes from the working household and labor demand from the enterprises and the government. The labor market is not at equilibrium and unemployment is caused by structural factors like skill mismatch, geographical reallocation of enterprises, and by inflexible labor contracts. Wages can be higher than the implicit equilibrium value because inflexible labor contracts adjust more slowly than inflation and productivity changes.

The market of goods and services is at equilibrium where the total GDP and imports equals consumption of the households, housing investment, investment in equipment and factories by the enterprises, government expenditures and exports. Our model does not describe the changes in inventories.

The money market is at equilibrium, demand for money includes the transactional motive which moves in the same direction to the volume of real transactions of the economy, the GDP and the speculative motive which moves in the opposite direction to the nominal interest rate. The supply of money is controlled by the central bank with its monetary policy. As private banks receive deposits and lend to the public, the total money stock is computed as the multiplication of the multiplier of money by the monetary base.

The foreign exchange market is described by a flexible exchange rate system. Exports will translate in supply of foreign currency and imports in demand for foreign currency. Flows of financial and direct investment will depend on the relative return in Japan and in the rest of the world and they become a source of supply or demand of foreign currency depending on the sign and size of arbitrage rate described before. As

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Japan has important assets denominated in foreign currencies, large amount of income from investment also add to the supply of foreign currency.

Dynamics of the Model

The dynamics of the model are commanded by the demographic transition, capital accumulation, growth in the rest of the world, expectations about prices, wealth allocation and economic policy.

Population natural growth depends on birth and death. Immigration and emigration are assumed unchanged in our simulations. Birth depend on the number of fertile women and the total fertility rate (TFR), which in turn, depends on the economic conditions described basically by real wages and employment relative to the cost of education. The number of deaths is calculated using the mortality rates by age provided by the Ministry of Health Labor and Welfare. Death rates have been kept constant along the path.

Physical capital grows with private and public investment, and falls with depreciation. Public investment depends on fiscal policy and private investment is described by equation (9) and also depends on the direction of the foreign direct investment because companies may prefer to allocate factories in other countries where the profit rate is higher.

Expectations

Both the household and the enterprises use in their calculations the future expected values of some variables like the inflation rate, nominal and real interest rate, variations in the exchange rate, future taxes etc. For example the household needs to forecast the future real wages, taxes, pension also the real interest rate when solving its utility maximization problem. The household is also an investor who decides how to allocate its total assets in the country and abroad. For this it needs to forecast the differential of interest rates, the differential of risks and the possible capital gains or losses that may result from changes in the value of the currency. Notice that all these variables are included in the intertemporal budget constraint. The enterprises need to forecast the expected real interest rate, the risk rate and the growth rate of profits to decide the level of investment.

Modeling expectations is very difficult, we have chosen a variation of static expectations for real wages, taxes and interest rate. Though we do not impose a Ricardian equivalence condition in the model, we assume the household thinks that the future income tax rates will rise in the future when the net debt of the government to GDP ratio is rising with respect to the average of the rest of the world. The working

household believes that social security contribution rate will be higher if the deficit of the social security fund is growing. The real interest rate considered by the household is the same as the considered by the enterprises, and it is a permanent adjustable real rate. The permanent interest rate is 2% in the benchmark and economic agents adjust it each year only partially according to the changes in the market.

Investors take into account the expected variations in the value of the currency when deciding on how to allocate their wealth in Japan and in the rest of the world. We assume the expected return of foreign investment is determined by the differential return in the current year. This means investors are supposed to believe that the future trend of returns will move in the same direction as in the current year. In any case investors are assumed to review their expectations using the observed values for each of the passing year.

Social security transfers from the government are adjusted annually according to the inflation rate and the number of beneficiaries which is commanded by the demographic structure of the population.

The monetary policy can affect the interest rate what in turns impacts investment, the value of the yen and the volume of exports. As other economic agents follow the target inflation and interest rate, they also make decisions accordingly.

The fiscal policy affects the economy in the present year but also in the future, as the higher the level of the debt, the higher and closer the tax hikes will be. Specific policies on child and education support have an impact on the population dynamics. The level of public works affect the level of demand in the present and in the future as investment becomes capital accumulation.

3. Simulations

We put together the equations of the model described in section 2 (the Mathematical Appendices section 2 lists all variables and parameters, section 3 lists all equations), and using the same estimations for most of the parameters of the model used in Bardas (2011) we performed the simulations using the Mathematica software. As our benchmark year is 2012, the exogenous variables correspond to 2012 and some parameters were recalibrated to reproduce the equilibrium of this year.

As explained in Bardas (2011), we use very simple econometric techniques to estimate several parameters of the model using time series data. For those parameters which no empirical information exists like for example the preference rate of discount and the preference towards leisure, their values were estimated by the calibration

method. This method uses the exogenous and endogenous variables of the model for one or two years and solves for the unknown parameters. We refer to Bardas (2011) section 8, for the details of the estimated parameters and exogenous variables. The Mathematical Appendices include a summary of the final results.

We performed three simulations to analyze the long term performance of the Japanese economy under the monetary and fiscal stimulus designed by Abenomics. We also study the effect of the consumption tax hike expected for 2014 and 2015. Structural reform, the third arrow of the Abenomics is not considered in the simulations.

Simulation 1 generates the benchmark to which the Abenomics (simulations 2 and 3) will be compared. It considers a mild expansionary monetary policy which seeks to achieve “price stability” defined as keeping the inflation rate within the range 0%~0.5% annual rate which proves to be a consistent range with the potential growth rate of the economy which is close to 2%. This simulation does not include the increase in public investment nor the consumption tax hike.

Simulation 2 considers a monetary policy under implementation by the BOJ since April 2013. This policy tries to achieve and keep the inflation rate close to a 2% per year.

Simulation 3 tries to replicate the second and third arrow of the Abenomics as announced so far. It includes the monetary policy of simulation 2, an expansionary fiscal policy and the consumption tax hike. The level of public investment is increased by ¥10 trillion (¥3 trillion executed in 2013 and ¥7 trillion in 2014), thus from ¥16 trillion in fiscal year 2012 rises to ¥19 trillion in 2014, and ¥23 trillion in 2015. Then, the level of investment is kept constant 13 years, and slowly reduced to its original level of ¥16 trillion. The government also goes ahead with the consumption tax hike raising the tax rate to 8% in 2014 and to 10% in 2015.

Assumptions of the simulations

a. Global growth

In the face of the slowdown of the Chinese economy and the difficulty in finding a quick resolution to the delicate situation of the European economy, we have chosen a relatively conservative assumption about the world GDP long term real growth prospects. The annual growth rate starts close to 3% similar to the observed in 2012. The growth rate improves very slowly along the path reaching 4% in the year ten and continues on the same pace of slow recovery.

b. International prices and interest rates and risks

International prices are assumed to move in the same direction as global growth because continuous economic expansion put pressure on global resources and prices

rise. Although the recent development of shale gas in the United States and the existence of large reserves in China and other countries raise hopes of more abundant and cheaper supply of energy, environmental concerns about the hydraulic fracturing technology still remain. Therefore we use the estimations of the relation between global growth, international inflation and world interest rates as in Bardas (2011).

Of course, there are many external events that may threaten a smooth expansion in the world economy. First the territorial disputes with China, which would affect the flow of trade among the two countries. Second, the civil war in Syria, the volatile situation in Egypt, and the unsolved nuclear issue between Iran and the West, may lead to unexpected rises in the international prices of oil. However, we assume none of these events have a big enough impact to deviate growth and international prices from the smooth path described above.

c. Energy policy

Although the government is pushing to restart at least some of the nuclear reactors, we assume the government is unable to do it and keeps the present energy mix (fossil fuels, nuclear, renewable) = (88%, 2%, 10%). This has an important effect on the volume of imports of oil, natural gas and coal, which is needed to produce energy and makes the trade balance stay in red in the future path and puts pressure on the value of the currency.

d. Social security

In the simulations, we keep all social security programs as in the present, with all benefits being adjusted upwards by inflation and responding to the changing population and economic conditions. For example, unemployment benefits adjust by wage changes and the number of recipients is also adjusted according to the level of unemployment.

Simulations Results

Our overall assessment of Abenomics is that expansionary monetary and fiscal policies do have a short and medium term positive effect on economic growth, produces a fiscal surplus (Graph 17) and reduces the debt to GDP ratio (Graph 18). The huge monetary stimulus of simulation 2 can in fact achieve the targeted 2% inflation rate (Graph 1) and growth rate higher than the potential rate which is close to 2% (Graph 3). However in the long term the growth rate falls below its potential and it may turn even negative. There is no one single factor that can explain the fall in the growth rate. However, as we understand, a set of factors come together to produce a slowdown in the household consumption which being the most important component of final demand is the final cause of the growth slowdown in the long run.

Abenomics cause growth rates to soar to 4%~5%. The quantitative easing needed

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to keep the 2% inflation rate, induce investors to engage in yen carry trade operations, by which they send their funds abroad where they can get higher returns. The fall in the JPY stimulates exports and employment increases. The unemployment rate is seen to below its natural rate which for Japan is close to 3.5%~3.8% (Graph 2). The lower value of the JPY (Graph 7) produces a temporal positive wealth effect as the return of the foreign assets is higher if repatriated. Higher employment and the positive wealth effect produce high growth. At the same time the government achieves an overall surplus by 2018, the debt of the government starts falling too.

However without the implementation of the structural reforms, the potential growth rate stays close to 2% and then the economy overheats. By the eighth year the BOJ has to reverse the quantitative easing and the yen carry trade operations come to an end. Besides, with the debt falling the risks implicit in the high government disappear the JPY strengthens and exports lose steam. The wealth effect disappears and the total net assets of the household do not grow as fast as before. As growth slowdown companies reduce the number of workers and the expansion of total salaries paid slows down too. Demographic factors would start playing its role: as the real wage rate growth less quickly than in the benchmark, the total fertility rate follows a lower path, population fall more rapidly what also becomes a negatively effect on consumption growth (Graph 9). Finally, the growth rate of the economy falls to zero. While exports may continue growing (Graph 11) a falling consumption will drag investment down too (Graph 10).

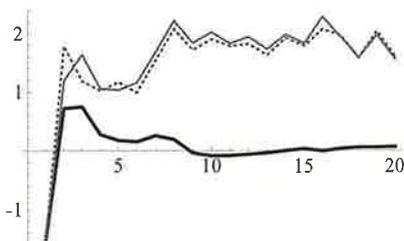
The following set of 18 graphs gathers the most important result of the simulations. The result for other variables can be provided if requested.

Bold line → Simulation 1: monetary policy pursues 0~0.5% inflation rate

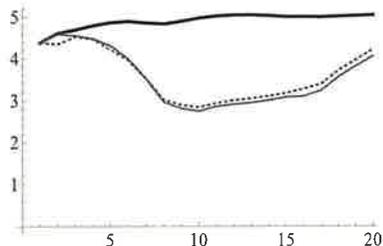
Thin line → Simulation 2: monetary policy pursues 2% inflation rate

Dotted line → Simulation 3: monetary policy +public works + consumption tax hike

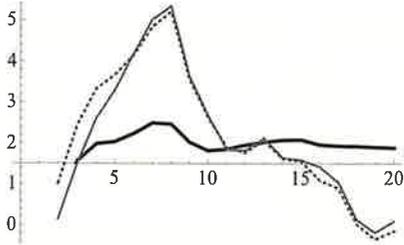
Graph 1: Inflation rate



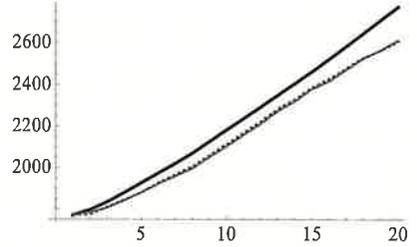
Graph 2: Unemployment rate



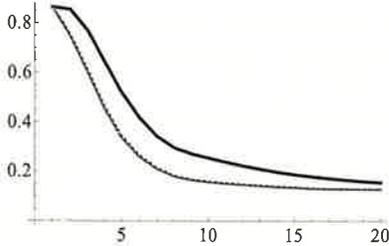
Graph 3: Growth rate



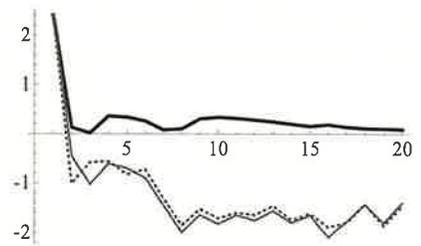
Graph 4: Real wage rate



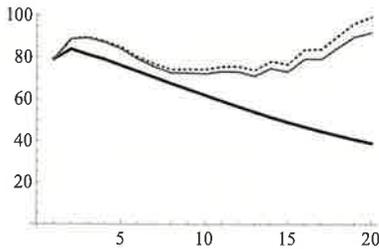
Graph 5: Nominal interest rate



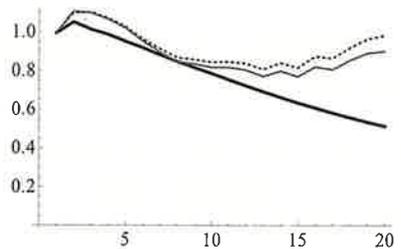
Graph 6: Real interest rate



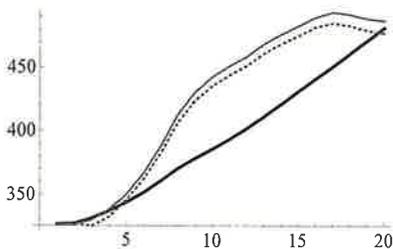
Graph 7: Nominal exchange rate



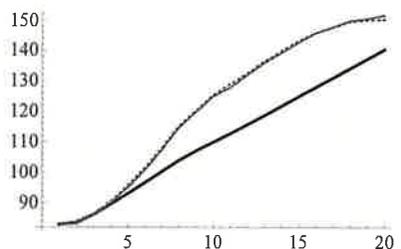
Graph 8: Real exchange rate



Graph 9: Real consumption

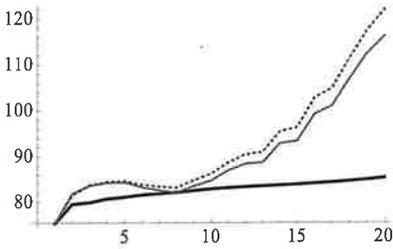


Graph 10: Real investment

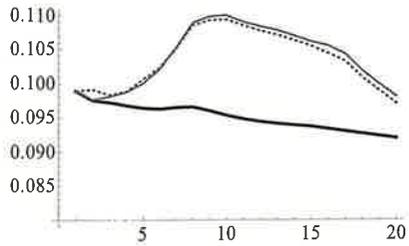


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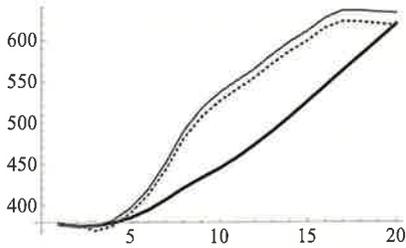
Graph 11: Real exports



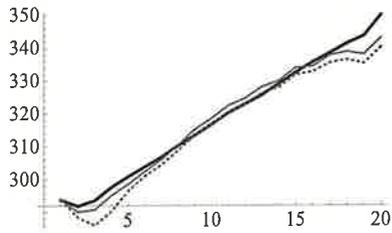
Graph 12: Employment



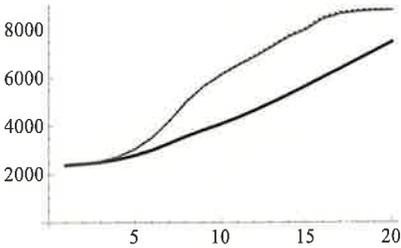
Graph 13: Consumption per capita (workers)



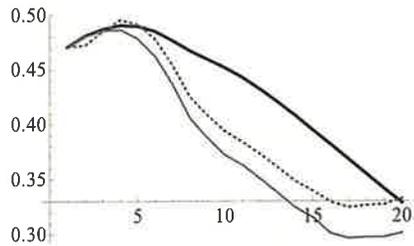
Graph 14: Consumption per capita (pensioners)



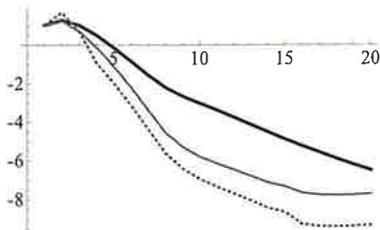
Graph 15: Household's net assets



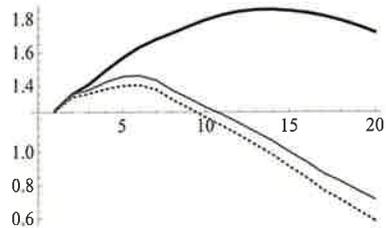
Graph 16: Social Security deficit



Graph 17: $\frac{\text{Deficit}}{\text{GDP}}$ %



Graph 18: $\frac{\text{Net Debt}}{\text{GDP}}$

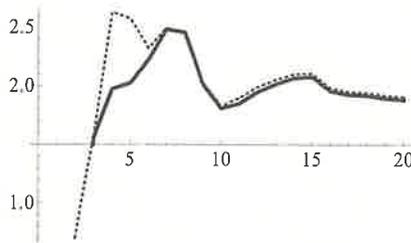


Notes: unemployment rate, growth rate, inflation rate, interest rates are in percent per year. The real wage rates and the household's net assets are expressed in JPY at constant prices of 2012.

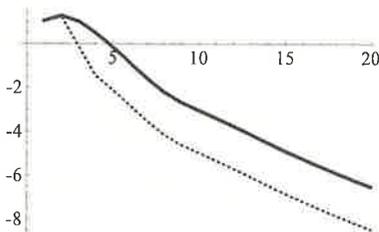
Moderate monetary policy with consumption tax hike

Finally we provide three more graphs which compare the benchmark monetary policy considered in simulation 1 (complete line) with a policy which includes also the consumption tax hike goes as planned, 8% in 2014 and 10% in 2015, (dotted line). These last graphs describe the path for the GDP growth rate, the deficit of the government and its debt to GDP ratio. It can be observed that the increase in the consumption tax would produce a fiscal surplus sooner than without it, growth would not be badly hurt and the debt to GDP ratio would start falling later on. If growth enhancing structural reforms were implemented further tax hikes could be avoided.

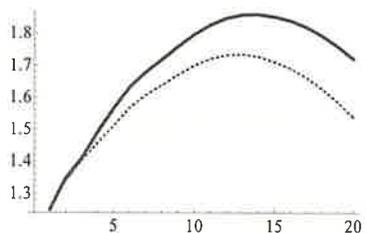
Graph A: Growth rate



Graph B: $\frac{\text{Deficit}}{\text{GDP}} \%$



Graph C: $\frac{\text{Net Debt}}{\text{GDP}}$



4. Conclusions

Under moderately optimistic conditions in the international markets, we test the effect of a large scale quantitative easing monetary policy and an expansion in public works as designed by the Abenomics on the performance of the economy and the debt of the government. According to our results, in the short and medium term the economy grows fast, and the debt to GDP ratio falls. However, if structural reforms are not implemented quickly enough, the economy overheats, becomes unstable and economic growth disappears. Instability is produced mainly by the yen carry trade operations in which investors sell their financial domestic assets and send their money abroad where rates of return are higher. Because the BOJ is buying those assets with the quantitative easing policy, the nominal interest rate remains very low and the real interest rate manages to stay negative for some time. This stimulates investment and consumption and as the economy overheats, the quantitative easing policy has to be suspended to keep inflation out of control, the yen carry trade operations reverse, the yen rises and exports slowdown. Falling employment means a lower labor income and consumption growth becomes negative pulling down the whole economy.

According to our simulations, if structural reforms cannot come quickly enough to raise the potential growth rate, keeping the economy stable and growing close to the 2% potential rate requires price stability that is an inflation rate of 0%~0.5%. Therefore, the monetary stimulus as designed by the present authorities is excessive. The consumption tax hike that would be executed in 2014 and 2015 does not have long lasting negative impact in the economy and the government reaches a surplus sooner than without it. However without structural reforms, the debt to GDP ratio remains high if compare to other countries.

So, can Abenomics revive the Japanese economy? We think that yes it can. However the government should speed up the national discussion required to achieve a consensus about the appropriate structural reforms which are essential to lift the potential long term growth rate of the economy. The effective implementation of these reforms is equally important. If this cannot be done quickly enough, the monetary stimulus must be readjusted to a more moderate one. In general terms, our conclusions are similar to the International Monetary Fund (August, 2013) and the Sri Mulyani Indrawati,(May, 2013) Managing Director of the World Bank, which are cautiously optimistic and recommends a prompt implementation of structural reforms.

Our model has to be improved in many ways and the estimation of some variables of the model has to be reviewed. The Abenomics is probably causing a shift in the

economic agents' expectations, what may affect some of our estimations like the consumption function, the investment function, and the decision making about holding national or foreign assets. As the economic agents digest the change caused by Abenomics and new data reveals their decisions the above functions could be reviewed. With respect to the theoretical model, our analysis respect to private investment includes its three main components: residential investment, inventories and enterprises investment in machinery and equipment in only one function, the investment function (equation 9). As the consumption tax is to be increased, in particular, the developments in the housing market could have an important impact in the economy, depending on how the tax hike is implemented. Therefore, it would be desirable to separate these components and include a housing market and a model for inventories which could provide better insight on the short term business cycle. From the above, the results of the simulations of our study should be taken as a contribution to understand the direction the economy may take under the Abenomics and less attention should be paid to the absolute level observed in the variables.

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Mathematical Appendices

In section 1 of this appendix we explain how the path for the economy is computed. In section 2 we summarize the main variables of the model, in section 3 the equations and in section 4 the estimated values for the parameters and exogenous variables at the first year of the benchmark (2012). Section 5 provides the equilibrium result for selected endogenous variables of the model for the year 2012. Section 6 describes the optimization problem of the households and the enterprises. For more detailed explanation of the theoretical model we refer to Bardas 2011.

1. Method of computation of the path of endogenous variables

The model has 81 endogenous variables, 50 exogenous or/and lagged endogenous variables and 44 parameters. The computation of the expected path for the variables of the model is computed following the following three steps.

STEP 1: parameters and exogenous variables are estimated either by econometrical methods or calibration (for details we refer to Bardas, 2011)

STEP 2: Numerical values obtained from STEP 1 are introduced in equations of the model and the general equilibrium for the endogenous variables in period 2012 (benchmark period) is computed.

STEP 3: Using the equilibrium values of endogenous variables from STEP 2 and the equations of the dynamics of the model (population dynamics, capital accumulation, debt accumulation, etc.) the general equilibrium for the endogenous variables for the following periods are obtained.

2. List of variables and parameters

2.1. Endogenous variable (81)

a) *Production (14)*

Q_t : value of production (the subscript “ t ” indicates “period”)

Y_t : GDP

K_t : utilized capital stock

In_t : volume of imported inputs used in the production process

N_t : labor demand (hours)

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$Employed_t$: labor demand (workers)

Ω_t : overwork - irregular time

λ_t : technological level

χ_t : technological progress

I_t : volume of private investment

Π_t : annual level of profits

μ_t : requirement of imported inputs

D_t : depreciation of capital

$NGDP_t$: nominal GDP

b) The households, labor and population variables (20)

U_t : level of utility of the working household

U_t^p : level of utility of the retired household

Z_t : level of preference between national and imported goods

C_t : volume of consumption of the working household

C_t^p : volume of consumption of the retired household

C_t : total volume of consumption of the households

S_t : saving of households

ℓe_t : hours of leisure

ℓ_t : hours of labor supply by the representative household

ℓ_E : expected hours of employment in the future

$LFPF_t$: labor force participation (women)

$LFPM_t$: labor force participation (men)

$FLabor_t$: women labor force

$MLabor_t$: men labor force

LF_t : total labor force

H_t : total supply of labor (hours)

TFR_{t+1} : total fertility rate

$Birth_t$: total birth

$Death_t$: total deaths

UN_t : unemployed people

c) Exports and Imports and foreign investment (10)

X_t : total volume of imports

IMs_t : substitutable imported goods

IM_t : total volume of imports

TFI_t : total stock of financial investment,

TDI_t : total stock of direct investment,

IFA_t : net income from the total holding of foreign assets

FIG_t : income earned from foreign assets owned by the government

FI_t : financial investment in foreign assets

DI_t : foreign direct investment

F_t : capital flow.

d) *Money demand, expected growth, risk, inflation and unemployment rates (13)*

L_t : demand for money

u_t : unemployment rate

π_t : inflation rate

g_t : growth rate of the GDP

g_{Et} : expected growth rate of the GDP by investors

Δg_E : variations in the expected growth rate

ρ_t : profit rate of productive activities

σ_t : risk rate of productive investment in Japan

σ_{jnt} : risk rate of secured Japanese bonds

$a_{c,t}$: consumption to GDP ratio

$a_{i,t}$: investment to GDP ratio

$a_{g,t}$: government expenditure to GDP ratio,

$a_{x,t}$ export to GDP ratio

e) *Government (12)*

$Contax_t$: collection of consumption tax

$Cortax_t$: collection of corporate tax:

$Assetax_t$: collection of asset tax:

$Imtax_t$: collection on tariffs:

$Inctax_t$: collection of the income tax, inhabitant tax and social security

$Taxes_t$: total taxes:

SSC_t : social security contributions

$UBEN_t$: unemployment benefits

Tr_t total transfers to working household

$Transfers_t$ total transfers to households and enterprise

ψ_t : government net debt to nominal GDP ratio

$\Delta Debt_t$ borrowing by the government

f) *Prices (12)*

P_t : average price level of the domestic and imported goods

P_{kt} : price of capital stock

w_t : nominal wage rate (¥ per hour worked)

Δw_t : change of the wage rate

w_E : is the expected average wage rate

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r_t : annual real interest rate

r_{Et} : expected annual real interest rate

e_{xt} : real exchange rate for exported goods

e_t : real exchange rate for imported goods

P_t : price level of domestic production

i_t : annual nominal interest rate

E_t : the nominal exchange rate (yens per units of foreign currency)

2.2. Exogenous and lagged endogenous variables (50)

a) Production (3)

KS_t : total capital stock available

θ_t : operation rate of the capital stock

ζ % of idle capital which remains unused

b) The household (4)

A_t : is the net present value of all assets owned by the working household

B_t includes and the NPV of expected transfers from the government

g_s : average expected growth rate of the wage rate

g_p : is the expected annual growth rate of future pensions.

c) Government (23)

t_i^c : indirect tax rate (consumption, tobacco, alcohol, etc.)

t_i^p : social security premiums

t_i : income tax (includes the inhabitant tax)

ct_t : corporate tax rate

at_t : asset tax rate

τ_E : average expected tax rate on labor

t_M : average tariff rate

ub : unemployment benefit effective rate

Kg_t : capital stock of the government

Cg_t : volume of consumption by the government

Ig_t : volume of investment by the government

G_t : total volume services and investment of the government

Dg_t : depreciation of capital of the government

Trw_t : transfers to working household

$Troid_t$: transfers to retired household

$Trener_t$: transfers to the enterprises

$Debt_t$: gross debt of the government

$NDebt_t$: net debt of the government

GA_t : financial assets of the government
 FAG_t : foreign assets owned by the government
 α_H : % of de debt owned be Japanese investors
 α_{BOJ} : % of de debt owned be the BOJ
 α_F : % of de debt owned by foreign investors

d) BOJ and other monetary and financial variables (8)

π_{target} : inflation rate target set by the BOJ
 H_t : monetary base
 M_t : amount of money circulating in the economy
 k : multiplier of money
 cd : cash to deposits ratio held by the public
 res_b : reserve requirement ratio
 res : technical reserves ratio
 a_σ : incidence of risk on growth

e) Relations with the Rest of the world (8)

Y_{wt} : real GDP of the rest of the world (PPP values)
 P_t^w : international price index of goods competitive with Japanese production (USD)
 $P_{im,t}^w$: international price index of imported goods (USD)
 i_t^w : world interest rate on funds valued in USD
 ρ_t^w : profit rate in the rest of the world
 σ_t^w : risk rate of investment in the rest of the world
 ψ_t^w : government net debt to nominal GDP ratio of the rest of the world
 $TNFI_t$: total stock of financial net assets abroad of the nation

f) Population, labor, unemployment and expected growth rate (7)

Pop_t : total population
 Pop age_{it} : population of age “i”
 FertileW_t : total fertile women
 Immigration_t : net immigration from abroad
 death rate_{it} : death rate of population of age “i”
 u_n : natural rate of unemployment
 se : Self-employed workers ratio

2.3. Parameters (44)

a) Households (6)

ρ : intertemporal annual rate of discount of the working household
 ρ_p : intertemporal rate of discount of the retired household
 ε : indicates how much utility the household receives from leisure.
 T : remaining years of life of the working household

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n : remaining years of work of the working household

s : remaining years of life of the retired household

b) Production (9)

λ_0, α, β : production function

m imported inputs per unit of product

δ, δ_g annual depreciation rate of private and government capital

a investment function

x_a, x_b : technological progress

c) Demand for money (2) : k_1, k_2

d) trade and foreign investment (9) $n_0, n_1, n_2, m_0, \eta, z_0, z_1, d_0, d_1$

e) fertility rate function (2) f_0, f_1

f) expected growth function (1) : Γ

g) risk rate function (2) : γ_1, γ_2

h) labor force participation (6) : $lf_0, lf_1, lf_2, lf_3, lf_4, lf_5$

i) standard hours available for work (1) : hours

j) irregular labor (2) : Ω_a, Ω_b

k) wage adjustment (4) : $\omega_0, \omega_1, \omega_2, \omega_3$

3. List of equations (81)

a) Production (14)

production function: $Q_t = \lambda_t N_t^\alpha K_t^\beta$

production to GDP: $Y_t = \mu_t Q_t$

utilized capital stock: $K_t = \theta_t K S_t + K G_t$

imported inputs: $I n_t = m Q_t$

labor demand (hours) $N_t = \left(\frac{w_t}{P_t} \frac{1}{\mu_t \alpha \lambda} \right)^{\frac{1}{\alpha-1}}$

labor demand (workers) $Employed_t = \frac{N_t}{\text{hours } \Omega_t} 1000^4$

irregular work function: $\Omega_t = \Omega_a - \Omega_b (u - u_n)$

technological level: $\lambda_t = \lambda_{t-1} (1 + \chi_t + \text{random})^{\chi} \quad |\text{random}| < 0.001$

technological progress $\chi_t = x_a + x_b \left(\frac{\Pi_t}{\Pi_{t-1}} - 1 \right)$

investment function: $I_t = \frac{a \Pi_t}{r_{Et} + \sigma_t - g_{Et}} - \xi (1 - \theta) K_t$

enterprises profits: $\Pi_t = P_t Q_t - w_t N_t - E_t P_{im}^w m Q_t$
 imported input coefficient: $\mu_t = 1 - \frac{E_t (1+t_m) P_{im}^w}{P_t} m$
 capital depreciation: $D_t = \delta P_t K_t$
 nominal GDP: $NGDP_t = P_t Y_t$

b) Households (19)

utility and preference functions

Working households $U_t = \sum_{j=t}^{t+T} \frac{\text{Log}(C_j) + \varepsilon \text{Log}(\ell_e j)}{(1+\rho)^j}$

Retired households $U_t^p = \sum_{j=t}^{t+s} \frac{\text{Log}(C_j^p)}{(1+\rho_p)^j}$

Import-domestic goods $Z_t = \left[(Y_t)^\eta + m_0 (IMS_t)^\eta \right]^{\frac{1}{\eta}}$

consumption functions

Workers $C_t = \frac{1}{\Phi_w} \frac{B_t}{P_t (1+t'_t)} + \frac{1}{\Phi_w} \frac{w_t (1-\tau_t)}{P_t (1+t'_t)} \ell_t + \frac{\Psi_w w_E (1-\tau_E)}{\Phi_w P_t (1+t'_t)} \ell_E$

Retirees $C_t^p = \frac{1}{\Phi_p} \frac{1}{P_t (1+t_c)} \left(B_t + \frac{\Psi_p + 1}{\Phi_p} Troid_t \right)$

total consumption $C_t = C_t + C_t^p$

Saving: $S_t = w_t \ell_t + Tr_t + Troid_t + IFA_t + i_t \alpha_\mu Debt_t + \Pi_t - SSC_t - Taxes_t - P_t C_t$

time constraint: $H_t = \ell_t + \ell_e$

labor supply $\ell_t = \frac{\left\{ \frac{\Phi_w - \Psi_w}{\varepsilon} \left[\frac{(1-\tau_E) w_E}{(1-\tau_t) w_t} - \frac{(1+r)}{(1+\rho)} \right] \right\} H_t - \frac{B_t}{w_t (1-\tau_t)}}{\Psi_w \frac{(1+r)}{(1+\rho)} + \frac{\Phi_w}{\varepsilon} + 1}$

Where $\Phi_w, \Phi_p, \Psi_w, \Psi_p$ are defined as follows

$$\Phi_w = \frac{1+\rho}{\rho} \left[1 - \frac{1}{(1+\rho)^T} \right] \qquad \Psi_w = \frac{1+g_s}{r_E - g_s} \left[1 - \left(\frac{1+g_s}{1+r_E} \right)^{n-1} \right]$$

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$$\Phi_p = \frac{1 + \rho_p}{\rho_p} \left[1 - \frac{1}{(1 + \rho_p)^s} \right]$$

$$\Psi_p = \frac{1 + g_p}{r_E - g_p} \left[1 - \left(\frac{1 + g_p}{1 + r_E} \right)^{s-1} \right]$$

expected employment:

$$l_E = N_t$$

labor force participation:

$$\text{LFPF}_t = \text{lf}_3 + \text{lf}_4 \frac{w_t}{P_t} \frac{P_{t-1}}{w_{t-1}} - \text{lf}_3 u_t \text{ (female)}$$

$$\text{LFPM}_t = \text{lf}_0 + \text{lf}_1 \frac{w_t}{P_t} \frac{P_{t-1}}{w_{t-1}} - \text{lf}_2 u_t \text{ (male)}$$

female labor force:

$$\text{FLabor}_t = \text{LFPM}_t \times \text{MWPop}_t$$

male labor force:

$$\text{MLabor}_t = \text{LFPM}_t \times \text{MWPop}_t$$

total labor force:

$$\text{LF}_t = \text{FLabor}_t + \text{MLabor}_t$$

total labor force available time:

$$H_t = 24 \times 365 \times \text{LF}_t$$

total fertility rate:

$$\text{TFR} = \text{TFR} (\Delta \text{wages}, \Delta \text{education costs})$$

births

$$\text{Birth}_t = \text{TFR}_t \times \text{Fertile}W_t$$

deaths:

$$\text{Death}_t = \sum_r \text{death rates}_{i,r} \times \text{Pop age}_{i,r}$$

unemployed people

$$UN_t = \text{LF}_t - \text{Employed}_t$$

c) Exports and Imports and foreign investment (10)

export function:

$$X_t = n_0 Y_w^n e_x^{n_2}$$

import function:

$$\text{IMS}_t = \left[\left(\frac{1}{e_t} + \frac{m}{1 - e_t m} \right) m_0 \right]^{\frac{1}{1-\eta}} \text{ (substitutable goods)}$$

total imports:

$$\text{IM}_t = \text{IMS}_t + \text{In}_t$$

total private financial investment:

$$\text{TFI}_t = z_0 \text{TNFI}_t + z_1 (i^w_t - i_t + \sigma_t - \sigma_{w,t})$$

direct investment

$$\text{TDI}_t = d_0 \text{Assets}_t + d_1 (\rho_{w,t}^E - \rho_t^E)$$

income from abroad

$$\text{IFA}_t = \text{TFI}_t \times i^w_t + \text{TDI}_t \times \rho_{w,t}^E \text{ (private agents)}$$

$$\text{FIG}_t = E_t \times \text{FAG}_t \times i_t^w \text{ (government)}$$

foreign financial investment:

$$\text{FI}_t = \text{TFI}_t - \text{TFI}_{t-1}$$

foreign direct investment:

$$\text{FD}_t = \text{TDI}_t - \text{TDI}_{t-1}$$

net flow of capital:

$$F_t = \text{FI}_t + \text{FD}_t - \Delta \text{FAG}_t$$

d) Money demand, unemployment, inflation, growth, and risk rates (13)

demand for money:

$$L = \frac{Y^{k_1}}{i^{k_2}}$$

unemployment rate:

$$u_t = \frac{UN_t}{\text{LF}_t} 100 + u_N$$

inflation rate

$$\pi_t = P_t \div P_{t-1} - 1$$

growth rate $g_t = Y_t \div Y_{t-1} - 1$

expected growth rate and variations in growth rate

$$g_{Et} = g_{Et-1} + \Delta g_E$$

$$\Delta g_E = a_{ct} \frac{\Delta C_t}{C_t} + a_{it} \frac{\Delta I_t}{I_t} + a_{gt} \frac{\Delta G_t}{G_t} + a_{xt} \frac{\Delta X_t}{X_t} - a_\sigma \frac{\Delta \sigma_{Et}}{\sigma_{Et}} + Radom$$

Profit rate $\rho_t = \frac{\Pi_t - D_t - Cortax_t}{P_{kt} K S_t}$

Risk rates $\sigma_t = g_t + \sigma_{fn} + random$

$$\sigma_{fnt} = \sigma_{fnt-1} + \gamma_1 \left(\frac{Debt}{GDP} \Big|_{Japan} - \frac{Debt}{GDP} \Big|_{Row} \right) + \gamma_2 \frac{P_{kt}}{P_t} \frac{P_{t-1}}{P_t}$$

consumption to GDP ratio $a_{ct} = C_t \div Y_t$

investment to GDP ratio: $a_{it} = Inv_t \div Y_t$

gov. exp. to GDP ratio $a_{gt} = G_t \div Y_t$

export to GDP ratio $a_{xt} = X_t \div Y_t$

e) Government (12)

collection of consumption tax $Cortax_t = t_c^c P_t C_t$

collection of corporate tax: $Cortax_t = ct_t (\Pi_t - D_t)$

collection of asset tax: $Assetax_t = at_t Asset_t$

collection on tariffs: $Imtax_t = E_t t_M P_{imt}^w IM_t$

collection of the income tax, inhabitant tax and social security

$$Inctax_t = t_i (1 - t_i^p) w_t \ell_t + t_i (1 - ct_t) (\Pi_t - D_t)$$

total taxes:

$$Taxes_t = Cortax_t + Inctax_t + Cortax_t + Assetax_t$$

social security contributions

$$SSC_t = t_i^p w_t \ell_t$$

unemployment benefits

$$UBEN_t = ub w_t UN_t$$

transfers to working household

$$Tr_t = Tr w_t + UBEN_t$$

government transfers

$$Transfers_t = Tr_t + Troid_t + Trenter_t$$

Net debt to GDP ratio

$$\psi_t = NDebt_t \div NGDP_t$$

borrowing: $\Delta Debt_t = P_t G_t + Transfers_t - (Taxes_t + Imtax_t + SSC_t + FIG_t + Dg_t)$

f) Prices (9)

average price level

$$P_t = \frac{P_t Y_t + E_t (1 + t_M) P_{imt}^w IM_t}{Y_t + IM_t}$$

price of capital stock

$$P_{kt} = \frac{\Pi_t - D_t - Cortax_t}{K_t} \frac{1 + g_{Et}}{r_{Et} + \sigma_t - g_{Et}}$$

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wage rate
$$w_t = w_{t-1} \left(1 + \frac{\Delta w_t}{w} \right)$$

change in wage rate:
$$\frac{\Delta w_t}{w} = \omega_0 \chi + \omega_1 \text{ sticky } \pi_{t-1} + \omega_2 (u_{t-1} - u_N) + \omega_3 \left(\frac{Inv_{t-1} - 1}{Inv_{t-2}} \right)$$

sticky = 0.1 when $\pi_{t-1} < 0$ and *sticky* = 0.4 when $\pi_{t-1} > 0$.

expected wage rate
$$w_E = w_t$$

real interest rate:
$$r_t = \frac{i_t - \pi_t}{1 + \pi_t} \text{ (observed)}$$

real expected interest rate:
$$r_{E,t} = r_{E,t-1} + \Gamma (r_t - r_{E,t-1}) \text{ (expected)}$$

real exchange rate for exported good:
$$e_{x,t} = \frac{E_t P^w_t}{P_t}$$

real exchange rate for imported good:
$$e_t = \frac{E_t (1 + t_m) P^w_{im,t}}{P_t}$$

g) equilibrium equations

1) The market of goods & services:
$$Y_t + e_t IM_t = C_t + I_t + G_t + \frac{E_t}{P_t} X_t$$

2) Money Market:
$$\frac{M_t}{P_t} = \frac{Y_t^{k_1}}{i_t^{k_2}}$$

3) Foreign exchange market:
$$P_t X_t - e_t IM_t + IFA_t + F_t = 0$$

3.2 Definitions and other mathematical relations (9)

budget constraint of working and retired households

$$\sum_{j=t}^{t+T} \frac{(1+t_j^c) P_j C_j}{(1+r_E)^j} = A_t + \sum_{j=t}^{t+n} \frac{w_j \ell_j (1-\tau_j) + Tr_j}{(1+r_E)^j} + \sum_{j=t+1}^{t+T} \frac{Trold_j}{(1+r_E)^j}$$

$$\sum_{j=t}^{t+s} \frac{(1-t_j^c) P_j C_j^p}{(1+r_E)^j} = B_t + \sum_{j=t}^{t+s} \frac{Trold_j}{(1+r_E)^j}$$

NPV of the household's assets:
$$B_t = A_t + \sum_t^{t+n} \frac{Tr_j}{(1+r_E)^j} + \sum_{t+n+1}^T \frac{Trold_j}{(1+r_E)^j}$$

total net foreign assets of the nation:
$$TNFI_t = TFI_t + TDI_t + E_t FAG_g$$

total government expenditures:
$$G_t = Cg_t + Ig_t$$

total tax rate paid on labor
$$\tau_t = 1 - (1-t_t)(1-t_t^p)$$

government net assets:
$$NDebt_t = Debt_t - G\dot{A}_t$$

multiplier of money
$$k = (1+cd) \div (cd + res_b + res)$$

money stock:
$$M_t = k H_t$$

3.4 Equations for the dynamics of the model (11)

- 1) Capital stock accumulation: $KS_{t+1} = KS_t (1 - \delta) + I_t$
- 2) Capital stock accumulation: $Kg_{t+1} = Kg_t (1 - \delta_g) + Ig_t$
- 3) Population accumulation: $Pop_{t+1} = Pop_t + Birth_t - Death_t + Imigration_t$
- 4) Debt of the government: $Debt_{t+1} = Debt_t (1 + i_t) + \Delta Debt_t$
- 5) Asset of the Households: $Asset_{t+1} = Asset_t + Saving_t$
- 6) Financial Asset of the government: $GA_{t+1} = GA_t + E_t FAG_t$
- 7) Adjustment for inflation of government expenses:
- $$Cg_{t+1} = Cg_t (1 + \pi_t), \quad Ig_{t+1} = Ig_t (1 + \pi_t)$$
- $$Trw_{t+1} = Trw_t (1 + \pi_t), \quad Troid_{t+1} = Troid_t (1 + \pi_t)$$
- $$Trenter_{t+1} = Trenter_t (1 + \pi_t)$$
- 8) operation rate of capital stock: $\theta_t = \theta_{t-1} 0.9 \text{Log} \left(2 + \frac{t}{t+1} \right)$

4. Estimations of parameters and exogenous variables.

Parameters

- a) $(\rho, \rho^p, \varepsilon) = (0.015, 0.03, 2.2)$, $(T, n, s) = (42.6, 25, 8)$
- b) $(\alpha, \beta, \lambda_0, m, a) = (0.55, 0.45, 58.7, 0.075, 0.026)$,
 $(\delta, \delta_g) = (8.2\%, 4\%)$, $(x_a, x_b) = (0.0113, 0.104)$
- c) $(k_1, k_2) = (1.06, 0.423)$
- d) $(n_0, n_1, n_2) = (0.05, 0.655, 0.55)$ $(\eta, m_0) = (0.456, 0.16)$
 $(z_0, z_1) = (0.16, 15.4)$, $(d_0, d_1) = (0.008, 5.25)$
- e) $(f_0, f_1) = (2.63, 1.28)$
- f) $\Gamma = 0.003$
- g) $\gamma_1 = 0.11$, $\gamma_2 = 0.01$
- h) $(lf_0, lf_1, lf_2, lf_3, lf_4, lf_5) = (0.58, 0.04, 4 \times 10^{-4}, 0.37, 0.03, 3 \times 10^{-4})$
- i) hours = $8 \times (365 - 104 - 20)$
- j) $(\Omega_a, \Omega_b) = (0.99, 0.053)$
- k) $(\omega_0, \omega_1, \omega_2, \omega_3) = (0.4, 1.296, -0.81, 5.1)$

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Some exogenous variables in the benchmark first year (2012)

$KS_t = 2,428$ trillion JPY, $w_0 = 1,976$ JPY/ hour, $(\theta, \zeta) = (0.9, 0.1)$

$(A_t, B_t) = (4,009, 2,008)$ trillion JPY, $(g_s, g_p) = (1.6\%, 1.44\%)$

$(t_t^c, t_t^p, t_t, ct_t, at_t, t_m, ub) = (0.07, 0.26, 0.105, 0.165, 0.005, 0.06, 0.12)$

$(G, Tr_t, Troid_t, Trenter_t) = (56, 55, 73, 3.7)$ trillion JPY

$(Debt_t, GA_t) = (1, 095, 382)$ trillion JPY, $FAG_t = 1.61$ trillion USD

$(\alpha_H, \alpha_{BOJ}, \alpha_F) = (0.832, 0.098, 0.07)$

$\pi target : 2\%$, $Ht = 132$ trillion JPY, $(res_b, res, cd) = (0.8\%, 4\%, 0.1)$, $\sigma fin = 2\%$

$Ywt = 82$ trillion, $Ptw = 0.012$ USD, $Piwm t$ USD

$itw = 5.75\%$, $\rho tE = 9.75\%$, $\rho tw = 9.75\%$, $\rho tw = 5.4\%$, $\psi tw = 0.68$

$(TFI_t, TDI_t) = (108, 57)$ trillion JPY

$un = 3.8\%$, $gEt = 2\%$

$Pop_t = 127$ million, $Pop ageit$ and $FertileWt$ (provided upon request)

$se = 0.1371$

5. The equilibrium in 2012 of main endogenous variables

Inflation rate = 0%

Unemployment rate = 4.3%

Nominal interest rate = 0.8%

Real interest rate = 2.4%

Exchange rate = 1 USD = ¥80

GDP = ¥471 trillion

Salaries = ¥246 trillion

Consumption of the Households = ¥334 trillion

Private Investment = ¥77 trillion

Exports = ¥75 trillion

Imports = ¥80 trillion

Government consumption and investment = ¥53 trillion

Deficit of the Government = ¥50 trillion

Values of other variables can be provided if requested

6. The optimization problems

6.1. The working household

The representative working household receives happiness or utility U_t from consumption and leisure, and looks for the combination of consumption (C_t), leisure (ℓe_t) and labor (ℓ_t) which gives her the maximum wellbeing in its whole lifespan and in a way that is compatible with its life budget constraint. The time constraint says that all available time, H_t is distributed in labor and leisure. The utility function is the logarithm of a Cobb Douglas type function, where ρ is the intertemporal rate of discount and ε is a parameter that represents the preference towards leisure. Consider the household which expects to live T more years which is solving its optimization problem in period “ t ”.

$$\begin{aligned} \text{Max } U_t &= \sum_{j=t}^{t+T} \frac{\text{Log}(C_j) + \varepsilon \text{Log}(\ell e_j)}{(1+\rho)^j} & (I) \\ \text{s.t. } & \begin{cases} \ell_t + \ell e_t = H \\ \sum_{j=t}^{t+T} \frac{(1+t_j^c) P_j C_j}{(1+r_E)^j} = A_t + \sum_{j=t}^{t+n} \frac{w_j \ell_j (1-\tau_j) + Tr_j}{(1+r_E)^j} + \sum_{j=t+1}^{t+T} \frac{Trold_j}{(1+r_E)^j} \end{cases} \end{aligned}$$

The left side of the budget constraint includes the net present value of consumption, and the right side includes assets and the net present value of future expected income from labor and transfers from the government. The discount interest rate, r_E is the average permanent real interest rate as expected at the moment of the decision making in period “ t ”. The value of consumption is computed multiplying the volume of consumption by the price level P , and t_j^c is the consumption tax rate. A_t is the given total initial assets owned by the household. Labor income is computed as the wage rate multiplied by the total worked hours $w_j (1-\tau_j) \ell_j$, where τ_j is the tax rate on labor income (income tax, inhabitant tax and social security premiums). The household receives transfers from the government Tr_j (health, child subsidy, unemployment benefits, education subsidies, etc.) during its working years, and $Trold_j$ (health, pensions, care for the elderly, etc.) during its retirement years. All transfers are net of other non-labor taxes like the asset tax, income tax on dividends, etc. The solution to the above problem gives the consumption function and the labor supply.

6.2. The retired household

The retired household does not work and all the available time is assigned to leisure. Therefore, the optimization problem consists in the maximization of the utility from consumption C_t^r for the remaining of its life.

$$\begin{aligned}
 \text{Max } U_t^p &= \sum_{j=t}^{t+s} \frac{\text{Log}(C_j^p)}{(1+\rho_p)^j} & \text{(II)} \\
 \text{s.t. } \sum_{j=t}^{t+s} \frac{(1-t_j^c) P_j C_j^p}{(1+r_E)^j} &= B_t + \sum_{j=t}^{t+s} \frac{\text{Troid}_j}{(1+r_E)^j}
 \end{aligned}$$

The intertemporal rate of discount for the retired household is ρ_p . B_t is the net assets available when taking its decision at time “ t ”. The solution to the above problems gives the consumption of the retired household.

6.3. Profit Maximization by the enterprises

Enterprises use capital (both private and public, including land) represented by K_t , labor N_t , and imported raw materials to produce a volume of production Q_t . The imported component of inputs is computed as a fixed proportion m of the volume of production: mQ_t . Capital is fixed at the level reached in the previous year, so enterprises will choose the level of labor that maximizes profits. The profit maximization problem is described as follows.

$$\begin{aligned}
 \text{Max } P_t Q_t - w_t N_t - E_t P_{im}^w m Q_t & & \text{(III)} \\
 \text{s.t. } Q_t = \lambda_t N_t^\alpha K_t^\beta
 \end{aligned}$$

P_t is the price level of domestic production, E_t is the exchange rate and P_{im}^w is the international price of imported raw materials. The solution to the above problem will give the labor demand function.

6.4. National and Imported goods

Domestic consumers and enterprises decide how much of their purchases will come from domestic production and how much from abroad. It is assumed that they have a CES type preference function Z_t which is used to select among the two origins of the goods. Economic agents are assumed to minimize the cost of purchases subject to this preference function. The minimization problem is represented as follows.

$$\begin{aligned}
 \text{Min } P_t Y_t + E_t P_{im}^w \left(IM S_t + Y_t \frac{m}{1-e_t m} \right) & & \text{(IV)} \\
 \text{s.t. } Z_t = \left[(Y_t)^\eta + m_0 (IM S_t)^\eta \right]^{\frac{1}{\eta}}
 \end{aligned}$$

The solution to the above problem will give the total imports of substitutable goods.

