〈論　　文〉

Trade, Skills Differential and the Real Wage Gap in a model with endogenous level of education

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Abstract

We consider a simple 2-country 2-goods model of trade with 2 types of individuals who decide on how many hours to spend educating and training themselves. Those individuals who choose a high level of education may work in the more technologically advanced sector of production. We analyze the effects of trade liberalization on the patterns of productive specialization, on the level of education of the individuals, on wages and on their level of utility.

As trade liberalization does not produce a Pareto improving outcome in any situation, the government may intervene and implement compensation policies to prevent any individual to be worse off. Furthermore, we observe that under some conditions, world production and consumption may fall when individuals decide to obtain lower levels of education than under free trade.

As a consequence of trade liberalization, wage rate inequality will not necessarily worsen in the more technologically advanced or developed country, and it will not necessarily improve in the less developed country as predicted by the Hecksher-Ohlin model. We show how freeing the flow of trade leads to changes in the optimal level of education chosen by the individuals what affects the wage gap either widening or reducing it.

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Introduction

The economic literature has extensively studied the effects of trade liberalization on income inequality. While traditional Hecksher-Ohlin (HO) inspired models predict an increase in the level of inequality in the developed countries and a reduction in inequality in the developing countries, other models contradict this result. On the other hand, empirical evidence does not support unambiguously that inequality reduces with trade liberalization in the developing countries.

Consider the standard HO trade model with 2 countries (North and South), 2 goods and 2 factors (skilled and unskilled labor). Trade liberalization will induce countries to specialize in the production of the good in which it has a comparative advantage that is, the good that employs intensively the factor that is abundant in the country. As the South has a relatively large number of unskilled workers and the North has a large number of skilled workers, the wage of skilled workers will fall in the South and rise in the North, while the wage of unskilled workers will rise in the South and fall in the North. Thus inequality will rise in the rich North and fall in the poor South.

Wood (1994) uses a HO model with 3 types of workers: skilled, with basic education, and uneducated (who have comparative advantages in skill-intensive manufacturing, labor-intensive manufacturing, and agriculture) and assumes wages increases with skill. The effects of trade liberalization on the poor and rich country are similar to the predicted by the traditional HO model. However, in a country with a high proportion of medium-skill workers, and hence a comparative advantage in manufacturing, liberalization of trade can either increase or decrease wage inequality as workers in the middle of the wage distribution gain while those at the top and bottom lose.

Chun Zhu S., (2004) incorporates Northern product innovation and technology transfer in a HO model and shows that inequality can rise in both the North and the South. The creation of new, highly skill-intensive goods in the North raises the relative demand for skilled labor and hence raises inequality. As the creation of new goods causes the North to lose competitiveness in older goods, they migrate to the South. Since these older goods are skill intensive in the South, technology transfer increases the relative demand for Southern skilled labor, creating inequality.

While part of the literature (for example Katz and Autor, 1999) believes that that wage inequality in several advanced countries is mainly induced by technological change rather than by international trade, Feenstra R. and Hanson G. (2001) say that this conclusion is obtained in part, from a misreading of the data. In the empirical tests of the HO model, it is standard to assume that exports are produced entirely by combining domestic factors of production with domestically-produced intermediate inputs. However, this ignores the recent dramatic increase in foreign outsourcing and in the trade in intermediate inputs. The increase in the wage gap between skilled and unskilled workers stems from the fact that trade in inputs will induce a shift demand away from low-skilled activities, while raising relative demand and wages of the higher skilled, then inequality increases.

As Kremer and Maskin (2003) indicate, cross-country empirical evidence does not support undoubtedly the claim that trade liberalization reduces inequality in poor countries as suggested by HO models. Hanson, et al (1999) finds that during the 1980s the wage gap between skilled and unskilled workers in Mexico widened in spite of the implementation of trade liberalization policies. Robbins (1996) observes the same phenomenon in several Latin American countries. Lindert and Williamson (2001) argue that liberalization tends to be followed by increases in inequality, but causality is doubtful particularly since in several large countries (India, China, Russia and Indonesia) liberalization had been only partial. Milanovic and Squire (2005) provide a weak support for the hypothesis that reduction of tariff tends to be associated with an increase inter-occupational wage inequality.

In the discussion of the effects of trade liberalization on the wage differential, little attention has been paid to how the individuals change their optimal level of education as a consequence of the trade liberalization policy. This is important because this might change the wage gap between skilled and unskilled workers. Trade liberalization leads to a change in the relative price and therefore a change in the wage rates. However, this is just the initial direct effect. As workers respond to the stimulus of the change in the wage rates, they will adjust their level of education too. This indirect effect has been omitted by the literature, but as we show it may have an important effect on the wage differential. This adjustment of the level of education by the individuals will change their productivity what will exert a subsequent indirect effect on their wage rate. As a consequence of the direct and indirect changes we may have different possible patterns of specialization and also different possible changes in the skill differential.

We use a model with 2 countries, 2 goods and 2 types of individuals. Individuals, guided by their preferences will choose their level of education what in turn will determine their level of productivity and wage rate. As we will show, in our model it can not be guaranteed that labor is complete mobile among industries. This is because it may happen that individuals who choose a relatively low level of education may become "technologically constrained" and will not be able to work in the sector with higher requirements of education.

First of all, several factors are determining the comparative advantage and then the potential good in which it could specialize in if trade was liberalized. We show that, in general, each country will tend to specialize in the good in which it has a higher state of the technology and/or the sector in which its workers have a higher level of education. We also show that trade liberalization will not always produce an increase in the world production. As free trade may push individuals to reduce their level of education, production may fall. We pay special attention to the analysis of how the real wage gap between skilled and unskilled workers may widen or reduce depending on how trade liberalization affects the behavior of the individuals towards education. Finally, as free trade does not always produce a Pareto-improving outcome, we study how different kind of compensation policies could be implemented in order to prevent any individual to be worse off.

This paper is organized as follows. First, we describe the technology of production and the optimization problem of the individuals. Second, we characterize the equilibrium of the economy under autarky. Third, we analyze the effects of trade liberalization on the patterns of productive specialization on education and the real wage gap. Finally, we show how governmental compensation policies can improve the competitive outcome. The main text of this paper includes de basic results of the analysis and the Appendices the most important mathematical computations. More detailed computations not included in this paper can be provided if requested.

1. The Model

We consider a 2-country-2-goods-2-types of individuals model in which the level of productivity of workers is determined by the level of education they choose to acquire. Good-X and good-Y could be produced both in the Home Country (HC) as well as in the Foreign Country (FC). It is assumed that the sector-Y is "more technologically advanced" in the sense that the technology applied in the production process requires higher levels of education to its workers.

In each country a very large number of individuals of two different types exist: individuals-i with low skill and individuals-j with high skills. Differences in their preferences towards consumption and leisure is what makes individuals-j dedicate more time to educate and train themselves and achieve a higher productivity.

In our model perfect competition prevails, transportation costs are zero and barriers to trade are ignored. This implies that consumers will be indifferent between the domestically produced and imported versions of a good, when their prices are the same. We also assume that labor is completely immobile among countries, but, in general, mobile within the country. For simplicity we assume that the amount of labor is exogenous and that the individuals are identical in their innate ability to produce. Productivity of the individual is determined solely by his level of education.

1.1. The Sector of Production

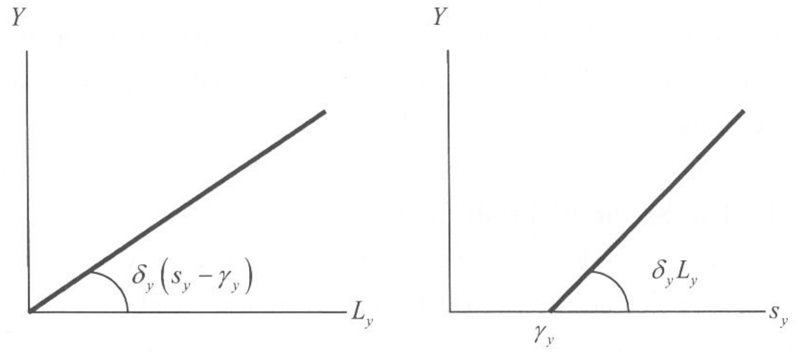
There are 2 sectors of production: the sector of good-X and the sector of good-Y. Each sector is composed by a large number of identical enterprises. The level of production is an increasing function of both labor and education. We assume that there is a minimum requirement of level of education in each sector of production: γ_x in sector-X and γ_y in sector-Y. Both γ_x and γ_y are exogenous positive parameters. We assume that requirements of education are higher in sector-Y: $\gamma_x < \gamma_y$. The production functions are the following

$$X = \delta_x (s_x - \gamma_x) L_x \quad \text{and} \quad Y = \delta_y (s_y - \gamma_y) L_y \tag{1}$$

X and Y stand for the volume of production; L_x and L_y for the amount of labor time employed and s_x and s_y for the level of education of workers of each sector (the subscript "x" and "y" indicate the sector of production). Individuals with different abilities are not required to work together and can work independently. δ_x and δ_y are positive parameters that indicate the state of technology.

Consider for example the production function of sector that produces good-Y. The volume of production Y can be depicted as an increasing linear function of the amount of labor L_y as in Figure 1 (graph at the left) or, for a constant level of labor, as a function of the level of education s_y the workers in that sector have acquired (graph at the right).

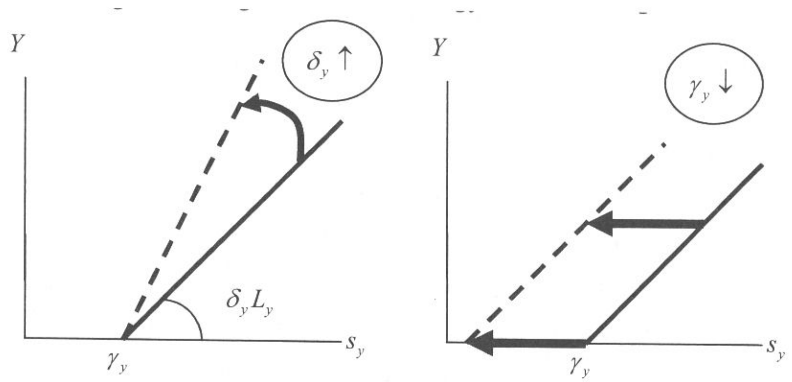
Figure 1: Two ways of representing the production function of good-Y



Left: as a function of labor　　*Right: as a function of education*

The position of the production function will change for different values of the two technological parameter δ_y and γ_y. A higher state of technology would be represented by either an increase of δ_y, a reduction in γ_y or a combination of them. Consider the production function as depicted in Figure 1 (right). We can see that an increase in parameter δ_y Figure 2 (left) makes the production function tilt on the opposite direction of the clock's hand. This means that for any specific value of the level of education the level of production is higher. On the other hand, a reduction in the minimum requirement of education, γ_y shifts the production function to the left and again, the level of production is higher for any level of education.

Figure 2: Higher state of technology in sector of good-Y



Left: an increase of δ_y Right: a reduction in γ_y

What will be the level of wage rate paid by each sector of production? To answer this question, we have that as total income equals total costs then $p_x X = w_x L_x$ and $p_y Y = w_y L_y$ will hold. Here, p_x and p_y are the prices of each good and w_x and w_y the income per unit of labor time or "wage rates" which can be computed as follows

$$w_x = p_x \delta_x \ (s_x - \gamma_x) \qquad \text{and} \qquad w_y = p_y \delta_y \ (s_y - \gamma_y) \tag{2}$$

1.2. The Optimization Problem of the Individual-i

There are two types of individuals who differ in their preferences towards consumption and leisure. The subscripts "i" and a "j" will be used to distinguish the two groups. In this section we describe the behavior of the individuals-i. The next section describes the behavior of individuals-j.

The individual maximizes his utility form consumption and leisure subject to his time and budget constraint. The volume of consumption of good-X by the individual-i is denoted by x_i, and the volume of consumption of good-Y by y_i and the amount of hours of leisure by ℓe_i. The individual is endowed with a total amount of time which it is normalized to 1. For simplicity labor (ℓ_i) is assumed to be exogenous. Then non labor time measured as the difference between the total endowment of time and labor time, $1-\ell_i$, will be allocated among education (s_i) and leisure. Therefore, the time restriction can be expressed as $1 = \ell e_i + s_i + \ell_i$

If the individual works in sector-X, his income per hour, w_i will be equal to the wage rate paid by that sector: $w_i = w_x$. If he works in sector-Y, then $w_i = w_y$. Income from labor is computed as $w_i \ell_i$. The lump sum tax paid by the individual is τ_i and the subsidy to education received is $z_i s_i$, where z_i is the subsidy rate per hour of education.

Assuming a constant elasticity of substitution (CES) U_i utility function, the optimization problem can be represented as follows

$$W_i = Max \quad U_i = \left[\left(x_i\right)^{\rho_i} + \alpha_i \left(y_i\right)^{\rho_i} + \beta_i \left(\ell e_i\right)^{\rho_i} \right]^{\frac{1}{\rho_i}} \\ s.t. \quad \begin{cases} 1 = \ell e_i + s_i + \ell_i \\ p_x\, x_i + p_y\, y_i = w_i\, \ell_i - \tau_i + z_i\, s_i \end{cases} \tag{3}$$

α_i, β_i and ρ_i are parameters such that $\alpha_i, \beta_i > 0$ and $-\infty < \rho_i < 1$. According to the specification of function U_i, the elasticity of substitution between the two consumption goods $\varepsilon_i = \dfrac{1}{1-\rho_i}$ is equal to the elasticity of substitution between any consumption good and leisure. When $0 < \rho_i < 1$, ε_i will be high, and when $\rho_i < 0$, ε_i will be low. After some computations we found that the optimal solution for x_i, y_i and ℓe_i is given by the following set of equations

$$x_i = \frac{w_i\,\ell_i - \tau_i + z_i\,s_i}{p_x + p_y \left(\frac{P}{\alpha_i}\right)^{\frac{1}{\rho_i - 1}}}, \qquad y_i = x_i \left(\frac{P}{\alpha_i}\right)^{\frac{1}{\rho_i - 1}} \tag{4}$$

$$\text{and} \qquad \ell e_i = x_i \left[\frac{1}{\beta_i p_x}\left(\frac{\partial w_i}{\partial s_i}\,\ell_i + z_i\right)\right]^{\frac{1}{\rho_i - 1}} \qquad \text{where} \quad P = \frac{p_y}{p_x}$$

The individual will choose to work in the sector of production which pays the higher wage rate for the level of education he has chosen. If he chooses to work at sector-X the level of education will be $s_{i/x}$ (the subscript "i/x" indicates that the individual-i works in sector-X) ; if he decides to work at sector-Y the level of education will be $s_{i/y}$. The formulas for $s_{i/x}$ and $s_{i/y}$ are as follows:

$$s_{i/x} = \frac{(1-\ell_i)\left[1 + \alpha_i \left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i - 1}}\right] + \left(\delta_x \gamma_x \ell_i + \frac{\tau_i}{p_x}\right)\left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i + \frac{z_i}{p_x}\right)\right]^{\frac{1}{\rho_i - 1}}}{\beta_i \left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i + \frac{z_i}{p_x}\right)\right]^{\frac{\rho_i}{\rho_i - 1}} + 1 + \alpha_i \left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i - 1}}} \tag{5}$$

$$s_{i/y} = \frac{(1-\ell_i)\left[1 + \alpha_i \left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i - 1}}\right] + \left(P\delta_y \gamma_y \ell_i + \frac{\tau_i}{p_x}\right)\left[\frac{1}{\beta_i}\left(P\delta_y \ell_i + \frac{z_i}{p_x}\right)\right]^{\frac{1}{\rho_i - 1}}}{\beta_i \left[\frac{1}{\beta_i}\left(P\delta_y \ell_i + \frac{z_i}{p_x}\right)\right]^{\frac{\rho_i}{\rho_i - 1}} + 1 + \alpha_i \left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i - 1}}}$$

Consider the case of non governmental intervention. As we assume $\gamma_x < \gamma_y$, a simple sufficient condition for $s_{i/x} < s_{i/y}$ is that

$$\frac{(1-\ell_i)-\gamma_y}{(1-\ell_i)-\gamma_x}\left(\frac{\delta_y}{\delta_x}P\right)^{\frac{\rho_i}{\rho_i-1}} < 1.$$

We show in section 2, equation (7) that, $\frac{\delta_y}{\delta_x}P > 1$ is a necessary condition for well defined levels of production. Therefore, when $\rho_i > 0$, then $s_{i/x} < s_{i/y}$ for any value of the parameters. When $\rho_i < 0$, $s_{i/x} < s_{i/y}$ may hold if $\frac{\delta_y}{\delta_x}$ is not too high. Well defined levels of education also requires $s_{i/x} < 1-\ell_i \Leftrightarrow \gamma_x < 1-\ell_i$ and $s_{i/y} < 1-\ell_i \Leftrightarrow \gamma_y < 1-\ell_i$

1.3. The Optimization Problem of the Individual-j

The optimization problem of the individuals-j can be represented in a similar way to the problem of individuals-i. All formulas are similar but the subscript "j" is used. Individuals-j may have different values for the parameters of the utility function α_j, $\beta_j > 0$ and $\rho_j < 1$. The volume of work, ℓ_j may be different too.

1.4. The Government

When the country opens to trade, the level of welfare of some individuals will increase while the level of welfare of others may fall. The government can intervene to implement compensations policies. It will collect taxes from those who have benefited by the liberalization policy and pay subsidies to those who have been hurt. The budget of the government can be represented as follows

$$\tau_i n_i + \tau_j n_j = z_i s_i n_i + z_j s_j n_j \tag{6}$$

n_i and n_j indicate the number of individuals of each type. τ_i and τ_j are the lump sum taxes (or "income transfer" if negative). z_i and z_j are the subsidy rate per unit of education time.

The compensation policies will be analyzed in section 7. From section 2 to section 6 it will be assumed that there is no governmental intervention.

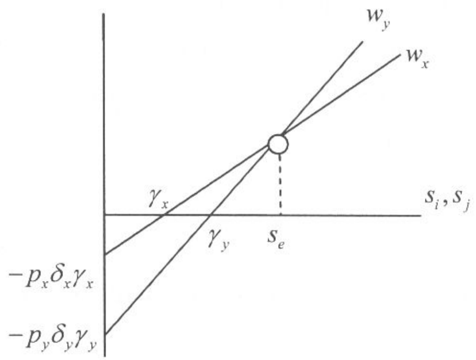
2. Where will the Individual Work in Autarky?

Figure 3 represents the wage rates w_x and w_y as a function of the level of education according to (2). The horizontal intercept is set to show that the minimum levels of education required in sector-Y is higher than in sector-X, that is $\gamma_x < \gamma_y$ as has been assumed. A necessary condition for both goods to be produced, is that the slope of function w_y is higher than the slope of function w_x: $p_y \delta_y > p_x \delta_x$. If $p_y \delta_y < p_x \delta_x$, line w_y would be below line w_x for any level of education s_i and s_j all individuals will prefer to work in sector-X. This will imply $Y = 0$ what cannot happen as both goods are desirable. Defining the relative price as $P = p_y / p_x$, the necessary condition for production of both goods to be positive, $X, Y > 0$ is:

$$P > \frac{\delta_x}{\delta_y} \tag{7}$$

The wage rate paid by the advanced sector, sector-Y, w_y will be higher than the wage rate paid by the less advanced sector-X only above a certain *"critical level of education"*, s_e.

Figure 3: The wage rate functions



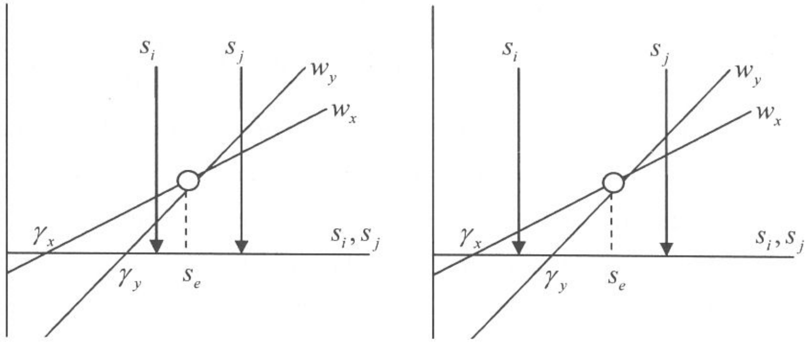
Observe from Figure 3 that the wage rates equalize at this *"critical level of education"*, s_e and its value can be computed as follows.

$$w_x = w_y \Leftrightarrow s_e = \gamma_y \frac{P\frac{\delta_y}{\delta_x} - \frac{\gamma_x}{\gamma_y}}{P\frac{\delta_y}{\delta_x} - 1} \tag{8}$$

Assume for example that $s_i < s_j$. The level of education of individuals-j ill fall at the right of the critical value s_e and the level of education of individuals-i will fall at the left. As represented in Figure 4, for individuals-i two cases are possible, $\gamma_y < s_i \leq s_e$ (graph at the left) and $\gamma_x < s_i \leq \gamma_y$ (graph at the right) In any case, looking for the highest wage rate, individuals-i would work in sector-X and individuals-j in sector-Y, and $w_x < w_y$ holds. Notice that in the graph at the right, individuals-i have chosen a low level of education and therefore are unable of working in sector-Y. This shows that labor is not completely mobile between the two sectors of production in any case.

Figure 4: The wage rate

Individuals-i will work in sector-X and individuals-j in sector-Y: 2 cases



The effect of an increase in the relative price on the level of education can be computed from the following equations.

$$\frac{\partial s_{i/x}}{\partial P}=-\frac{\rho_i}{1-\rho_i}\frac{1}{P}\frac{\left[(1-\ell_i)-\gamma_x\right]\alpha_i\beta_i\left(\frac{1}{\alpha_i}P\right)^{\frac{\rho_i}{\rho_i-1}}\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}}{\left[\beta_i\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}+1+\alpha_i\left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}\right]^2} \tag{9}$$

$$\frac{\partial s_{j/y}}{\partial P}=\frac{\rho_j}{1-\rho_j}\frac{1}{P}\frac{\left[(1-\ell_j)-\gamma_y\right]\beta_j\left(\frac{\delta_y\,\ell_j}{\beta_j}P\right)^{\frac{\rho_i}{\rho_i-1}}}{\left[\beta_j\left(\frac{\delta_y\,\ell_j}{\beta_j}P\right)^{\frac{\rho_j}{\rho_j-1}}+1+\alpha_j\left(\frac{P}{\alpha_j}\right)^{\frac{\rho_j}{\rho_j-1}}\right]^2}$$

As $\rho_i<1$, $\rho_j<1$, $1-\ell_i>\gamma_x$, and $1-\ell_j>\gamma_y$ hold,

$$\frac{\partial s_{i/x}}{\partial P}>0\Leftrightarrow\rho_i<0 \quad\text{and}\quad \frac{\partial s_{j/y}}{\partial P}>0\Leftrightarrow\rho_j>0 \tag{10}$$

Consider the individual-j. As ℓ_j is an exogenous variable, from the time constraint equation we have that $\frac{\partial \ell e_j}{\partial P} = -\frac{\partial s_j}{\partial P}$. When the elasticity of substitution ε_j is low ($\rho_j < 0$ and $|\rho_j|$ high) then $\frac{\partial \ell e_j}{\partial P} > 0$. Why would the individual react in this way? Consider the substitution and income effects. As the individual-j works in sector-Y, when P rises, his real wage rate[1] will initial increase. As the opportunity cost of leisure is the real wage rate, the individual will try to reduce ℓe_j: this is the substitution effect. The income effect makes the individual increase ℓe_j: as the wage rate increases, he feels richer and will try to expand leisure. As the income effect is more important than the substitution effect the individual ends up increasing ℓe_j and reducing s_j. When $0 < \rho_j < 1$ the income effect is small then, $\frac{\partial \ell e_j}{\partial P} < 0$.

3. The Relative Price Under Autarky

We assume that the set of parameters is such that under autarky the individuals-i work in sector-X and the individuals-j work in sector-Y. Then, the total hours worked in each sector can be computed as $L_x = \ell_i n_i$ and $Ly = \ell_j n_j$, where n_i and n_j indicate the number of individuals of each type. The total supply of each good are X and Y, the demand of good-X from each individual-i is x_i and from individual-j is x_j. Demand for good-Y are y_i and y_j. Therefore, the equilibrium conditions

1 The real wage rate is computed dividing the nominal wage by a price index. The price index increases but less than the nominal wage rate, then the real wage falls. We show how the price index is computed in section 6.

at market of good-X and of good-Y are expressed as follows.

$$X = x_i n_i + x_j n_j \tag{11-a}$$

$$Y = y_i n_i + y_j n_j \tag{11-b}$$

According to the Walras' law, one of the above equation is redundant and the equilibrium value of P can be determined by, for example, (11-b). Define the excess supply function $A\ (P)$ as

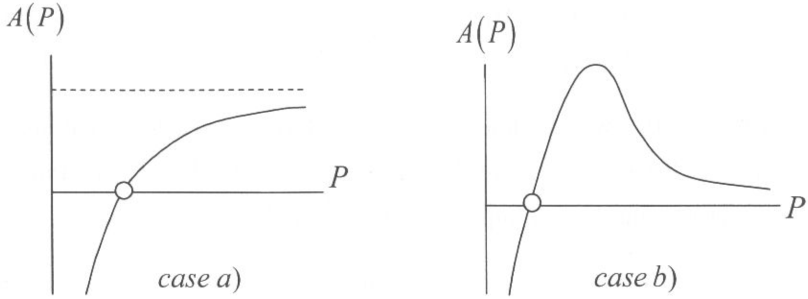
$$A\ (P) = Y - (y_i n_i + y_j n_j) \tag{12}$$

And then, using (4) and (5), $A\ (P)$ transforms into an implicit function of P:

$$A(P)=\frac{\delta_y \ell_j n_j\left[\left(1-\ell_j\right)-\gamma_y\right]}{\beta_j\left(\dfrac{\delta_y \ell_j}{\beta_j}P\right)^{\frac{\rho_j}{\rho_j-1}}+1+\alpha_j\left(\dfrac{P}{\alpha_j}\right)^{\frac{\rho_j}{\rho_j-1}}}-\frac{\delta_x \ell_i n_i\left[\left(1-\ell_i\right)-\gamma_x\right]\left(\dfrac{P}{\alpha_i}\right)^{\frac{1}{\rho_i-1}}}{\beta_i\left(\dfrac{\delta_x \ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}+1+\alpha_i\left(\dfrac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}} \tag{13}$$

As represented in Figure 5, the shape of $A\ (P)$ depends on the sign of ρ_i and ρ_j. Case a) occurs when i) ρ_i, $\rho_j > 0$ or ii) $\rho_i < 0$ with $\rho_j > 0$. Case b) occurs when i) ρ_i, $\rho_j < 0$ or ii) $\rho_i > 0$ with $\rho_j < 0$. In any case there is a unique equilibrium level of P measured at the horizontal intercept where $A\ (P) = 0$.

Figure 5: The equilibrium relative price under autarky



4. What Happens when Countries Open to Trade?

For the Foreign Country (FC) we will have a similar set of relations (1)-(13) and we use a superscript asterisk "*" to denote the FC. For example, the levels of production will be represented by X^* and Y^*, the autarkic relative price by P^*, etc.. Let's assume that at equilibrium the Home Country (HC) has a comparative advantage in the production of good-Y, then

$$P < P^* \tag{14}$$

We compute the partial derivatives of A with respect to the different parameters to investigate the conditions under which $P < P^*$ holds. The results of our computations for the HC are the following:

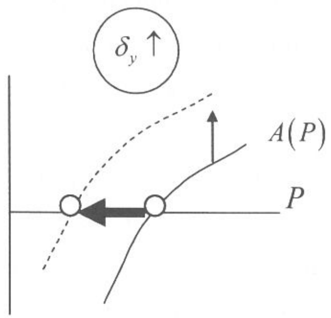
$$A_{\delta_y},\ A_{\gamma_x},\ A_{a_i},\ A_{\beta_j},\ A_{n_j} > 0 \text{ and } A_{\delta_x},\ A_{\gamma_y},\ A_{a_j},\ A_{\beta_i},\ A_{n_i} < 0 \tag{15}$$

And we will have similar results for the FC.

We can see how, for example, a higher parameter δ_y makes curve A shift upwards, what and the equilibrium relative price will be lower.

(Figure 6).

Figure 6: The determinants of the comparative advantage



What is determining the comparative advantage? The number of individuals of each type, the state of technology and also preferences determines which country will have a comparative advantage in which product. We study this issue below.

i. The Two Countries Differ in the Number of Individuals

Consider the extreme case in which the two countries are identical in technology and preferences but differ in the number of individuals of each type. Relation (14) would occur when the number of skilled workers (individuals-j working in sector-Y) is relatively more abundant in the HC: $n_j/n_i > n^*_j/n^*_i$ This is inferred from $A_{n_j} > 0$ and $A_{n_i} < 0$ and $A^*_{n_j} > 0$ and $A^*_{n_i} < 0$. As a higher ratio n_j/n_i makes curve $A(P)$ shift upwards, the horizontal intercept (which indicates the equilibrium autarkic price P) will be lower. On the other hand a lower n^*_j/n^*_i will make $A^*(P^*)$ shift downwards, which produces a higher level of the autarkic price P^*.

ii. The Two Countries Differ in the state of the Technology

Assume now that countries differ only on the state of technology. $P < P^*$ will hold for example if: $\delta_y > \delta^*_y$, while $\delta_x = \delta^*_x$, $\gamma_x = \gamma^*_x$ and $\gamma_y = \gamma^*_y$. In this case the HC is technologically relatively more efficient than the FC in the production of good-Y because for equal level of education the HC is capable of producing more of good-Y (and less of good-X) per hour worked.

On the other hand, assume $\gamma_y < \gamma^*_y$, while $\gamma_x = \gamma^*_x$, and $\delta_x = \delta^*_x$ and $\delta_y > \delta^*_y$. This makes the HC relatively more efficient in the production of good-Y because for a lower γ_y the production function rotates counter clock as explained in Figure 2.

Of course preferences also affect the direction of the comparative advantage, and (15) would give a hint of how differences in parameters determine $P < P^*$.

iii. The two countries differ in the level of education

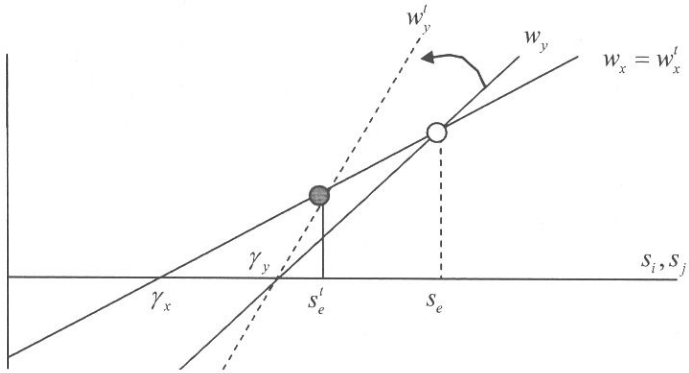
Assume the while number of individuals of each type and technology is the same in the HC and the FC, the level of education of the individuals of type-j is higher (or/and the level of education of individuals-i is lower) in the HC. It is easy to show that the graph A will always be located above A^* what means the HC will have a comparative advantage in good-Y.

Summarizing, the HC will have a comparative advantage in the production of good-Y when the number of individuals producing this good is larger, when their level of education is higher on when technology in this sector is higher.

4.1. The Terms of Trade

As countries open to trade, goods are expected to flow from where they are produced cheaper to where they are more expensive. The HC would export good-Y and the FC good-X. The flow of commerce would occur at a world price P^t such that $P < P^t < P^*$. Trade liberalization will make the relative price increase in the HC (and reduce in the FC). Therefore, the wage rate paid by sector-Y will increase for any given level of education (see (2)). As shown in Figure 7, this can be represented by a counter-clock rotation of line w_y to $w^t{}_y$ (the "t" superscript indicates "free trade") fixed at point γ_y in the horizontal axe. The line w_x remains unchanged because as the model only determines the relative price level $P^t = p^t{}_y / p^t{}_x$ and $p^t{}_x$ is consider as a constant.

Figure 7: Trade liberalization: line w_y rotates to $w^t{}_y$



The *"critical value under free trade"* $s^t{}_e$ is the level of education at which $w^t{}_x = w^t{}_y$ We will refer to $s^t{}_e$ in section 4.3. Notice that $s^t{}_e < s_e$ (see equation (8)).

$$w_x^t = w_y^t \Leftrightarrow s_e^t = \gamma_y \frac{P^t \dfrac{\delta_y}{\delta_x} - \dfrac{\gamma_x}{\gamma_y}}{P^t \dfrac{\delta_y}{\delta_x} - 1} \tag{16}$$

However, the final pattern of specialization will not settle until the individuals adjust their levels of education. We dedicate the following section to this problem.

4.2. The Change in the Level of Education

When trade is liberalized, the relative price and the wage rates will change, then the individuals will adjust their optimal level of education. We have 2 possible outcomes. 1) *Complete Specialization* (CS) : each country produces only the good in which has a comparative advantage. 2) *Partial Specialization* (PS) : the country produces both goods and exports the good in which has a comparative advantage. Section 4.3 describes the conditions under which each of these cases may occur.

Which case will prevail after trade liberalization will depend on the initial position of s_i and s_j, and the direction and size of the movement. While the position of s_j is always above the critical value s_e ($s_e < s_j$), there are two possible relevant initial positions for s_i: $\gamma_x < s_i < \gamma_y$ and $\gamma_y < s_i < s_e$. The direction of the movement of s_i and s_j will depend on the sign of ρ_i and ρ_j as indicated by (10).

Figure 8 and 9 describe the possible reaction of the individual-i and Figure 10 the reaction of individual-j. Figure 8 represents the case in which the initial position is such that $\gamma_x < s_i < \gamma_x$; Figure 9 when $\gamma_y < s_i < s_e$. The vertical complete lines labeled "s_i" and "s_j", indicate the position of the level of education of individuals under autarky. The broken vertical lines will indicate the possible shifts of the level of education.

Consider Figure 8 which shows the behavior of s_i. According to (10), when $\rho_i < 0$, s_i may shift to positions ①, or to position ③ if the change is big enough. In the case $\rho_i > 0$, s_i will fall to position ②.

In Figure 9 it is shown the relevant possible new position of s_i after trade liberalization which are ④ (when $\rho_i < 0$), and ⑤ and ⑥ (when $\rho_i > 0$).

Figure 8: Initial position $\gamma_x < s_i < \gamma_y$: the 3 possible shifts of s_i

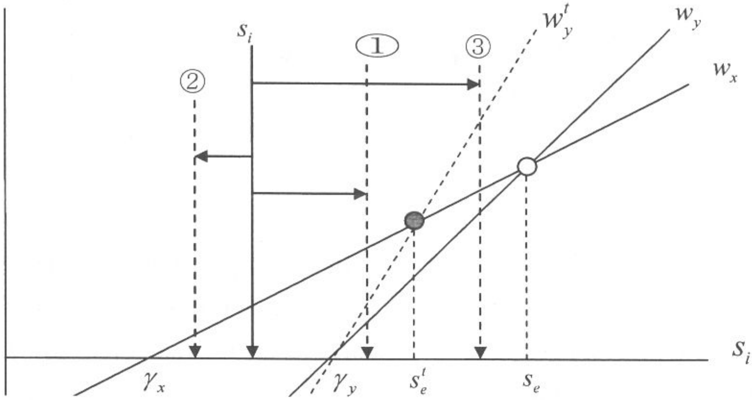
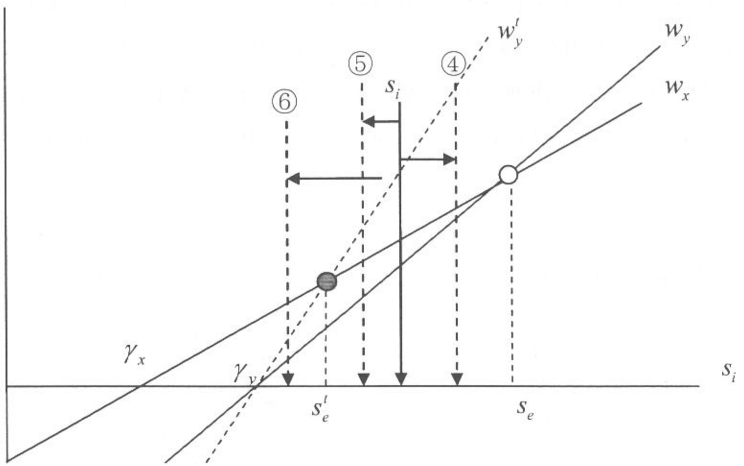


Figure 9: Initial position $\gamma_y < s_i < s_e$: the 3 possible shift s_i



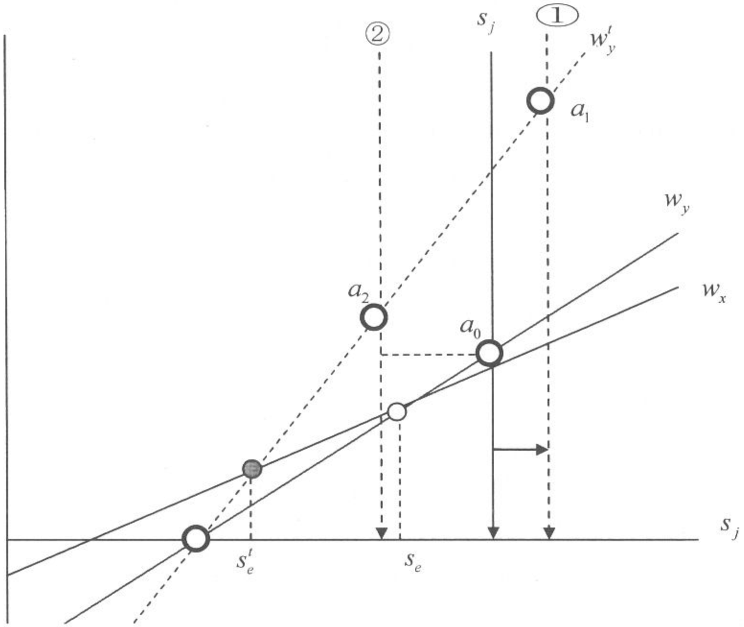
As trade liberalization pushes the relative price upwards, and when $\rho_i < 0$ individuals-i may increase the level of education to a higher level than the critical value s^t_e. (case ③ and ④). Above this level they would shift to the production of good-Y where a higher wage rate is being paid. A higher wage would induce them to reduce in a certain amount the level of education. If this reduction is not big enough, they will remain working in sector-Y where the wage rate is higher. On the other hand when $\rho_i > 0$, the individuals will reduce education. If the shifts are big enough like in ② and ⑥ the individual will continue producing good-X. If the reduction is small like in ⑤ they will shift to the production of good-Y.

Then consider how the individual-j adjusts his level of education as a reaction to trade liberalization. As shown in Figure 10, s_j may increase (to position ①) or fall (to position ②).

Consider first the case the individual increases his level of education to position ①. The liberalization of commerce increases the wage rate of the individual (from a_0 to a_1) and makes him fell richer, and more willing to increase leisure, this is the income effect. However the substitution effect which makes the price of leisure (the wage rate) higher is more important. As he reduces leisure and increases his level of education he will be able to consume more. According to (10), s_j increases when $\rho_j > 0$.

In the case the level of education falls to position ②, the income effect is more important than the substitution effect and the individual tries to expand leisure what makes him reduce education. According to (10), s_j falls when $\rho_j < 0$. It is not difficult to see that trade liberalization will never make the individual reduce the level of education s_j below the critical level s^t_e. This is because s_j reduces only while the wage rate is increasing and this does not happen when $s^t_j < s^t_e$. Trade liberalization makes the wage rate rise from the initial position a_0 to a_2.

Figure 10: The 2 relevant shifts of s_j



From the above graphs it can be inferred that world production may fall. This is because while we have assumed the hours of labor exogenous, the level of education of both types of individuals can fall in both the HC and the FC. Of course, a reduction in the volume of production does not necessarily mean a reduction in the utility of the individuals because leisure is expanding.

4.3. Patterns of Specialization

4.3.1. Complete Specialization (CS)

"CS" occurs when trade liberalization induces all individuals to work in the production of the good in which the country has a comparative advantage (sector-Y in the HC). The level of education of the individuals, s^t_i and s^t_j (the superscript "t" indicates the situation under free "trade") will surpass the critical level s^t_e which is defined by equation (16). Accordingly

$$s^t_e < s^t_i \quad \text{and} \quad < s^t_e < s^t_j \tag{17}$$

From (10) a necessary condition for CS to occur is $\rho_i < 0$. s_i may shifts to position ③ of Figure 8, or either to position ④ or ⑤ of Figure 9. Besides, s_j may shift to positions ① or ② of Figure 10. CS will occur more easily when for example when the distance $|\gamma_y - \gamma_x|$ is small and when $s^t_i - s_i$ is high which will occur when ℓ_i is low because then the individual has long hours available to allocate into education. This can be checked this from the Appendices section 1.2..

Notice that although complete specialization occurs and both types of individuals work in the production of the same good, their level of education may be different, and as a result the wage rates will be different too.

4.3.2. Partial Specialization (PS)

"PS" occurs when after trade liberalization the country continues producing both goods and exports the good in which it has a comparative advantage. In the HC the levels of education of the individual are such the following condition holds:

$$s^t_i < s^t_e < s^t_j \tag{18}$$

PS will occur when s_i shifts to any of the positions ①, ② of Figure 8, and ⑥ of Figure 9. It can be shown that PS will occur more easily when: a) the distance $|\gamma_y - \gamma_x|$ is high, b) s_i is relatively low and 3) the change in s_i due to liberalization is not important enough.

5. Effects of Trade Liberalization on Utility

Under certain conditions, the policy of subsidies will allow the trade liberalization to become Pareto-improving, that is, no individual is worse off with the policy and at least some individuals are better off. In order to know the conditions under which a Pareto improving situation occurs, we must compare the level of utility of the individuals under autarky W_i, W_j and under unrestricted trade $W_i^t W_j^t$. The necessary and sufficient condition for the trade liberalization to be a Pareto improving situation is

$$\frac{W_i^t}{W_i} \geq 1 \quad \& \quad \frac{W_j^t}{W_j} \geq 1 \tag{19}$$

As analyzed in more detail in the Appendices section 1.3., with trade liberalization individuals-j will always be better off but individuals-i may be better off only in the case of complete specialization and only if P^t is high enough. In the following Table we summarize the effects of trade liberalization on the level of utility of the individuals.

Table 1: Effects of liberalization on utility

	SIGN OF THE CHANGE IN UTILITY ACCORDING TO THE TYPE OF SPECIALIZATION	
	Complete Specialization	Partial Specialization
Individual-i	− if P^t is low + if P^t is high	−
Individual -j	+	+

6. Effects of Trade Liberalization on the Real Wage Rate Gap

Define the *nominal* wage rate ratio in the case of autarky (I) and under unrestricted trade (I^t) as follows

$$I = \frac{w_j}{w_i} \qquad \text{and} \qquad I^t = \frac{w_j^t}{w_i^t} \tag{20}$$

By definition I > 1. How will trade liberalization affect the *real* wage rate differential? To compute the real wage rates we must consider a price index for each individual. Let PI_i and PI_j represent the price indices for individuals i and j respectively:

$$PI_i = \frac{p_x^t x_i + p_y^t y_i}{p_x x_i + p_y y_i} \qquad \text{and} \qquad PI_j = \frac{p_x^t x_j + p_y^t y_j}{p_x x_j + p_y y_j} \tag{21}$$

The *real wage rate index* (RWI) measures the real wage differential:

$$\text{RWI} = \frac{I^t}{I}\frac{PI_i}{PI_j} \tag{22}$$

When RWI > 1 wage inequality widens, and when RWI < 1 wage inequality reduces and may even reverse.

From (21) we can also find the following interesting and useful relation for the quotient of the price indices PI_i/PI_j

$$\frac{PI_i}{PI_j} = \frac{x_i + P^t y_i}{x_i + P\, y_i}\frac{x_j + P\, y_j}{x_j + P^t y_j} \geq 1 \Leftrightarrow \frac{y_i}{x_i} \geq \frac{y_j}{x_j} \tag{23}$$

As trade liberalization increases relatively the price of this good the above says that the price index is higher for the individual who consumes more of good-Y.

As we explain below, there are two effects operating in the change in the wage rates 1) *the direct effect* of the change in the relative price and 2) *the indirect effect* of the changes in the level of education.

To see more clearly these two effects, it is illustrative to consider the simple case of $\varepsilon_i, \varepsilon_j \to 1$ ($\rho_i, \rho_j \to 0$). This is case the in which the utility functions transform into Cobb-Douglas type functions and the indirect effect is exactly zero.

6.1. The Cobb-Douglas ($\rho_i, \rho_j \to 0, \rho_i^*, \rho_j^* \to 0$)

When $\rho_i, \rho_j \to 0$ equations (5) become

$$s_i\big|_{\rho_i \to 0} = \frac{(1-\ell_i)(1+\alpha_i) + \beta_i \gamma_x}{1+\alpha_i+\beta_i} \tag{24}$$

and

$$s_j\big|_{\rho_j \to 0} = \frac{(1-\ell_j)(1+\alpha_j) + \beta_j \gamma_y}{1+\beta_j+\alpha_j} \tag{25}$$

Notice that s_i and s_j are independent of the relative price, then, trade liberalization will not affect neither the levels of education nor the level of production. The individuals will continue working in the same sector as under autarky and CS will occur. Changes in the wage rates will be due solely to the direct effect of the change in the relative price. It is not difficult to show that (22) becomes:

$$\mathrm{RWI}\big|_{Cobb} = \frac{(1+\alpha_j)P^t}{(1+\alpha_i)P} \frac{P+\alpha_i P^t}{P+\alpha_j P^t} \tag{26}$$

where the "Cobb" subscript indicates the Cobb-Douglas case.

As $P < P^t$ it is easy to see that $\mathrm{RWI}\,|_{Cobb} > 1$ in any case. This will mean that wage inequality increases in the HC and reduces in the FC. As the HC has a comparative advantage in the production of good-Y which requires higher skills than in the production of good-X the result is on line with the Hecksher-Ohlin prediction. How does this result change when we consider CES utility functions? We investigate this in the next section.

6.2. The Case of $\rho_i, \rho_j \neq 0$

We show that wage inequality may increase or reduce depending on how workers adjust their level of education. In order to perform this analysis we will not study the whole range of conditions under which wage inequality reduces or expands. Instead we will only consider two extreme cases to show that both cases are possible. First of all notice that simple sufficient conditions for wage inequality increase or decrease could be synthesized as presented in Table 2.

Table 2: Sufficient conditions for increase and reduction in wage inequality

	Wage inequality increases (RWI >1)	Wage inequality reduces (RWI <1)
Condition 1	$\mathrm{I}^t > \mathrm{I}$	$\mathrm{I}^t < \mathrm{I}$
Condition 2	$PI_i / PI_j > 1$	$PI_i / PI_j < 1$

Condition 1 indicates whether the *nominal* wage rate index increases ($\mathrm{I}^t > \mathrm{I}$) or reduces ($\mathrm{I}^t < \mathrm{I}$) due to trade liberalization. Condition 2 indicates who consumes more of what good. For example, when $PI_i / PI_j > 1$ the individuals-i are consuming relatively more of good-Y than individuals-j. As the price of Y is increasing relatively to X, individuals-i will see their purchasing power reduced more than individuals-j. On the contrary, when $PI_i/PI_j<1$ the individuals-i consume relatively less of good-Y than individuals-j.

As developed in detail the mathematical expression for RWI is different in the case of partial specialization and complete specialization and we present the detailed computation results in the Appendices section 6, but as we find out the set of specific conditions under which inequality increases or reduces are quite similar.

We summarize the results we have obtained in the Table 3.

Table 3: Effects of liberalization on the wage gap

inequality increases when → (RWI >1)	$\rho_i,\ \rho_j>0$ And $\frac{y_i}{x_i} \geq \frac{y_j}{x_j}$
inequality reduces when → (RWI <1)	$\rho_i,\ \rho_j<0$ with $\lvert\eta_i\rvert$ and $\lvert\eta_j\rvert$ high And $\frac{y_i}{x_i} < \frac{y_j}{x_j}$

Note: $\rho_i,\ \rho_j$ are the parameters of the utility functions and $\lvert\eta_i\rvert$ and $\lvert\eta_j\rvert$ are the elasticities of s_i and s_t with respect to the price level according to (27) (see detailed computations in the Appendices section 6)

$$\eta_i=\frac{ds_i}{dP}\frac{P}{s_i} \qquad \eta_j=\frac{ds_j}{dP}\frac{P}{s_j} \tag{27}$$

We can see that inequality will clearly increase when trade liberalization makes the individuals-i reduce their level of education (this happens when $\rho_i > 0$) and at the same time makes the individuals-j increase it (this happens when $\rho_j > 0$) if the individuals-i are consuming relatively more of good-Y. As consequence of this adjustments in the levels of education, individuals-i will receive a lower nominal wage rate, and the individuals-j a higher nominal wage rate what widens the nominal wage gap. Besides, as the individuals-i are consuming relatively more of the good in which the country has a comparative advantage (good-Y) which price is increasing, then, they are loosing more purchasing power of their wages.

Notice, that this case gives similar prediction as the Hecksher-Ohlin model. Trade liberalization has increased the wage inequality in the HC which has a comparative advantage in the production of the more technologically advanced sector of production.

However, we may also have an opposite result. Trade liberalization can induce a reduction in wage inequality in the HC even if it has a comparative advantage in the more technological advanced product. This happens when the individuals-j working in the advances sector are reducing its level of education (this happens when $\rho_j < 0$), while the individuals-i working the sector less technologically advanced, are increasing their level of education (this happens when $\rho_i < 0$). As consequence of this, individuals-i will receive a higher nominal wage rate. The individuals-j will also be able to improve their nominal wage rate but not too much because they are reducing their level of education at the same time. Now, if compared with individuals-i, the individuals-j are consuming relatively more of the good in which the country has a comparative advantage (good-Y) which price is increasing, then they are loosing more purchasing power of their wages.

7. Compensation Policies

While trade liberalization will not always be Pareto improving, we show in the Appendices section 3 and section 4, that the government can implement transfers policies that may, under certain conditions, allow all individuals to benefit from the trade liberalization policy. When the utility of individuals-i fall, the government should pay a compensation. For this the government will collect taxes from individuals-j the ones who are being benefited with the trade liberalization. We study to types of compensation policies: first a lump sum subsidy paid with a lump sum tax, and second a proportional subsidy to education to individuals-i paid with a lump sum tax collected from individuals-j. According to our results, both polices can produce a Pareto efficient outcome.

Furthermore, the second type of compensation policy which subsidies the level of education of the individuals-i may even make them increase their level of education in such a way that they may shift to the production of good-Y. Then the policy would conduce the country to complete specialization.

8. Conclusions

The main objective of this paper is to analyze how trade liberalization affects the patterns of specialization as well as the real wage gap between skilled and unskilled workers. The analysis focuses on how trade liberalization affects the wage rates of the individuals with different skills and how this makes the individuals change their optimal bundle of education and leisure time. We consider a simple 2-country-2-goods model of trade in which there are 2 types of individuals who decide on how many hours to spend on leisure and on education. We assume that one sector of production employs a higher level of technology so requires higher levels of education to its workers than the other sector. Different individuals will naturally choose different levels of education, and those who choose to acquire higher levels of education will work in the more technologically advanced sector and will also receive higher level of the wage rate.

Several factors are determining the comparative advantage and then the potential good in which it could specialize in if trade was liberalized. We show that, in general, each country will tend to specialize in the good in which it has a higher state of the technology and/or the sector in which its workers have a higher level of education. We also show that trade liberalization will not always produce an increase in the world production.

As trade liberalization occurs, the relative prices and the wage rates will change. This is the direct effect of the liberalization policy on wages. The indirect effect will include how the wage rate change as the individuals adjust their behavior to the new prices and then will decide on a different level of education. As they decide to increase or decrease their effort in education and training, their wage rate will experience a additional change. Then we measure how this direct and indirect changes affect the wage gap.. According to our results inequality does not necessarily worsen with trade liberalization in the more advanced country nor necessarily improves in the less developed country as predicted by the Hecksher-Ohlin model. For example, contrary to the predictions of the Hecksher-Ohlin model, the real wage gap may improve in the developed country in the following situation. Trade liberalization will make the home country specialize in the production of the more technologically advanced product when the level of education of the workers employed in that sector have a higher level of education than their peers in the foreign country. Then the home country has a comparative advantage in the product which employs an advanced technology. As trade and specialization occur, the relative price in the home country rises and the workers of the home country employed in the advanced sector will see their wage increased. This may induce them to either increase or decrease their level of education depending on how important is the so called substitution effect and the income effect. If leisure is very important to them, higher income levels may make them more willing to expand leisure and reduce hours of training and education. Trade liberalization gives these workers a higher wage rate, but as they are reducing their level of education, it is not as high as it could be if they did not change their level of education. If at the same time workers in the less advanced sector of production decide to expand their level of education, then the nominal wage gap will reduce. Finally, to consider the real wage gap we must compute the price indices of each type of workers. If for example the more educated individuals who are working in the technologically advanced sector of production are consuming more of the good which price has risen, then the real wage rate gap will reduce.

Finally, we suggest some directions for future research. First, the analysis of our model can be enriched considering labor supply as en endogenous variable. Secondly, more general results could be obtained using non linear specifications of the technological relation and even allowing for the two types of workers to be substitutes in a joint production. Finally, a dynamic model in which education is treated as human capital would reveal very interesting aspects about how the liberalization polices affect the pattern of specialization in both the short and the long run.

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Appendices[1]

1. Complete specialization (CS)

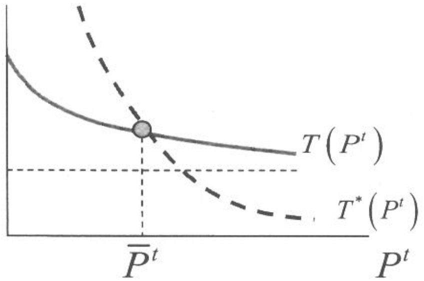
1.1. The terms of trade at equilibrium

The equilibrium value of P^t can be computed using (11-b). $X = 0$, $Y^* = 0$ and $Y = \delta_y (s_i - \gamma_y) \ell_i n_i + \delta_y (s_j - \gamma_y) \ell_j n_j$

$$Y = y_i n_i + y_j n_j + y^*_i n^*_i + y^*_j n^*_j \tag{A1}$$

y_i, y_j, y^*_i, y^*_j can be obtained using (4) (evaluated at P^t). Define the *excess supply function* of the HC as $T(P^t) = Y - (y_i n_i + y_j n_j)$ and the *excess demand function* of the FC as $T^*(P^t) = Y - (y^*_i n^*_i + y^*_j n^*_j)$. Equilibrium occurs at $T(P^t) = T^*(P^t)$ (figure A1).[2]

Figure A1: The equilibrium level of P^t in the case of CS



1 More detailed computations can be provided if requested.

2 The mathematical expressions of functions $T(P^t)$ and $T^*(P^t)$ are omitted because of space constraints but we can provide them if requested.

1.2. Effect on the Level of Education

Compute $s^t_i - s_i$ and $s^t_j - s_j$ for the HC:

$$s^t_{i/y} - s_{i/x} = \frac{\left[\beta_i\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}} + 1 + \alpha_i\left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}\right]^{-1}\beta_i\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}} N}{\beta_i\left(\frac{\delta_y\,\ell_i}{\beta_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}} + 1 + \alpha_i\left(\frac{P^t}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}} \tag{A2}$$

where

$$\begin{aligned} N = &\left[\left(1-\ell_i\right)-\gamma_x\right]\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}\left[1+\alpha_i\left(\frac{P^t}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}\right] \\ &+\left[\gamma_y-\left(1-\ell_i\right)\right]\left[1+\alpha_i\left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}\right]\left(\frac{\delta_y\,\ell_i}{\beta_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}} \\ &+\left(\gamma_y-\gamma_x\right)\beta_i\left(\frac{\delta_y\,\ell_i}{\beta_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}}\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}} \end{aligned}$$

$${}_i - s_j = \frac{\beta_j\left[\left(1-\ell_j\right)-\gamma_y\right]\left[1-\left(\frac{P^t}{P}\right)^{\frac{\rho_j}{\rho_j-1}}\right]\left(\frac{\delta_y\,\ell_j}{\beta_j}P\right)^{\frac{\rho_j}{\rho_j-1}}}{\left\{\beta_j\left(\frac{\delta_y\,\ell_j}{\beta_j}P^t\right)^{\frac{\rho_j}{\rho_j-1}}+1+\alpha_j\left(\frac{P^t}{\alpha_j}\right)^{\frac{\rho_j}{\rho_j-1}}\right\}\left\{\beta_j\left(\frac{\delta_y\,\ell_j}{\beta_j}P\right)^{\frac{\rho_j}{\rho_j-1}}+1+\alpha_j\left(\frac{P}{\alpha_j}\right)^{\frac{\rho_j}{\rho_j-1}}\right\}} \tag{A3}$$

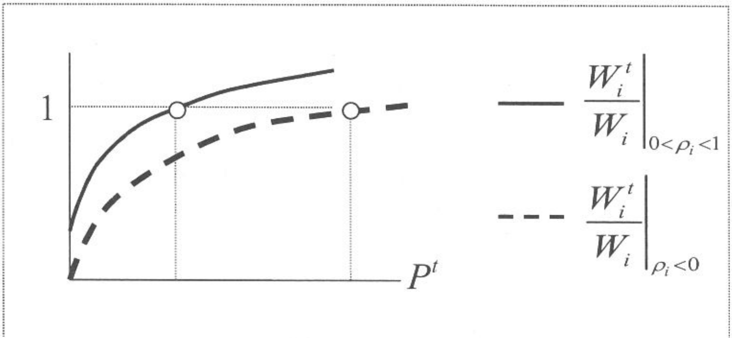
1.3. Effect on the Level of Utility

Compare the level of the indirect utility under free trade, W_i^t and under autarky, W_i:

$$\frac{W_i^t}{W_i}=\frac{\delta_y\left(1-\ell_i\right)-\gamma_y}{\delta_x\left(1-\ell_i\right)-\gamma_x}\left[\frac{\left(P^t\right)^{\frac{\rho_i}{1-\rho_i}}+\alpha_i\left(\frac{1}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}+\beta_i\left(\frac{\delta_y\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}}{1+\alpha_i\left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}+\beta_i\left(\frac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}}\right]^{\frac{1-\rho_i}{\rho_i}} \tag{A4}$$

From, Figure A2 $W_i^t > W_i$ if P^t is high enough

Figure A2: Utility under free trade and autarky in the case of CS



For the individual-j, $W_j^t > W_j\ \forall\ \rho_j$ as can be concluded from (A3)

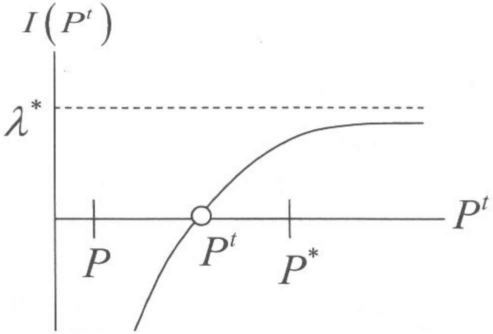
$$\frac{W_j^t}{W_j}=\left[\frac{\left(P^t\right)^{\frac{\rho_j}{1-\rho_j}}+\alpha_j\left(\frac{1}{\alpha_j}\right)^{\frac{\rho_j}{\rho_j-1}}+\beta_j\left(\frac{\delta_y\ell_j}{\beta_j}\right)^{\frac{\rho_j}{\rho_j-1}}}{P^{\frac{\rho_j}{1-\rho_j}}+\alpha_j\left(\frac{1}{\alpha_j}\right)^{\frac{\rho_j}{\rho_j-1}}+\beta_j\left(\frac{\delta_y\ell_j}{\beta_j}\right)^{\frac{\rho_j}{\rho_j-1}}}\right]^{\frac{1-\rho_j}{\rho_j}} \tag{A5}$$

2. Partial Specialization (PS)

2.1. The terms of trade at equilibrium

Using (11-b). we define the excess world supply function of good-Y as $I\ (P^t) = A\ (P^t) + A^*\ (P^t) = 0$ where A and A^* are the excess demand functions of the HC and FC respectively. (similar to equation (13) but evaluated at P^t)

Figure A3: The equilibrium of P^t in the case of PS



2.2. The Effect of Trade on the Level of Education

The expression for $s^t_j - s_j$ is the same as (A3), $s^t_i - s_i$ is as follows

$$s_i^t - s_i = \frac{\alpha_i \beta_i \left[(1-\ell_i) - \gamma_x \right] \left[\left(\frac{P^t}{P} \right)^{\frac{\rho_i}{\rho_i - 1}} - 1 \right] \left(\frac{\delta_x \ell_i}{\beta_i} \right)^{\frac{\rho_i}{\rho_i - 1}} \left(\frac{P}{\alpha_i} \right)^{\frac{\rho_i}{\rho_i - 1}}}{\left\{ \beta_i \left(\frac{\delta_x \ell_i}{\beta_i} \right)^{\frac{\rho_i}{\rho_i - 1}} + 1 + \alpha_i \left(\frac{P^t}{\alpha_i} \right)^{\frac{\rho_i}{\rho_i - 1}} \right\} \left\{ \beta_i \left(\frac{\delta_x \ell_i}{\beta_i} \right)^{\frac{\rho_i}{\rho_i - 1}} + 1 + \alpha_i \left(\frac{P}{\alpha_i} \right)^{\frac{\rho_i}{\rho_i - 1}} \right\}} \quad \text{(A6)}$$

As $P < P^t$, $\rho_i, \rho_j < 1$, $(1-\ell_i) > \gamma_x$, and $(1-\ell_j) > \gamma_y$ then $s^t_i < s_i \Leftrightarrow \rho_i > 0$ and $s^t_j < s_j \Leftrightarrow \rho_j < 0$. For the FC, as $P^* > P^t$, $s_i^{*t} < s^*_i \Leftrightarrow \rho^*_j < 0$ and $s_j^{*t} < s^*_j \Leftrightarrow \rho^*_j > 0$.

2.2. The effects of trade on utility

From the two equations below it is not difficult to show that, as $P < P^t$, then $W_i^t/W_i < 1 \ \forall \rho_i$ and trade. $W_j^t/W_j > 1 \ \forall \rho_j$:

$$\frac{W_i^t}{W_i} = \left[\frac{1 + \alpha_i \left(\frac{P^t}{\alpha_i} \right)^{\frac{\rho_i}{\rho_i - 1}} + \beta_i \left(\frac{\delta_x \ell_i}{\beta_i} \right)^{\frac{\rho_i}{\rho_i - 1}}}{1 + \alpha_i \left(\frac{P}{\alpha_i} \right)^{\frac{\rho_i}{\rho_i - 1}} + \beta_i \left(\frac{\delta_x \ell_i}{\beta_i} \right)^{\frac{\rho_i}{\rho_i - 1}}} \right]^{\frac{1-\rho_i}{\rho_i}} \tag{A7}$$

$$\frac{W_j^t}{W_j} = \left[\frac{\left(P^t\right)^{\frac{\rho_j}{1-\rho_j}} + \alpha_j \left(\frac{1}{\alpha_j} \right)^{\frac{\rho_j}{\rho_j - 1}} + \beta_j \left(\frac{\delta_y \ell_j}{\beta_j} \right)^{\frac{\rho_j}{\rho_j - 1}}}{\left(P\right)^{\frac{\rho_j}{1-\rho_j}} + \alpha_j \left(\frac{1}{\alpha_j} \right)^{\frac{\rho_j}{\rho_j - 1}} + \beta_j \left(\frac{\delta_y \ell_j}{\beta_j} \right)^{\frac{\rho_j}{\rho_j - 1}}} \right]^{\frac{1-\rho_j}{\rho_j}}$$

3. Compensation policy (lump sum tax-transfers)

Assume the government levies individuals-j with a lump sum tax ($\tau_j > 0$) and then uses the proceeds to pay a lump sum subsidy ($\tau_i < 0$) to the less educated individuals-i. The budget of the government becomes $\tau_i n_i + \tau_j n_j = 0$. We compare the level of utility under autarky (W_i, W_j) with the level under free trade when compensations are performed (W_i^t, W_j^t). The necessary and sufficient condition for the utility of both types of individual to increase is the following.

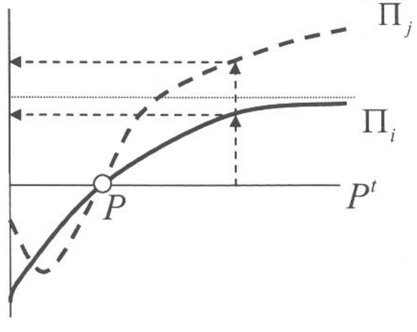
$$\frac{W_i^t}{W_i} \geq 1 \ \ \& \ \ \frac{W_j^t}{W_j} \geq 1 \Leftrightarrow \Pi_i \leq \frac{\tau_j}{p_x^t} \leq \Pi_j \tag{A8}$$

$$\text{Where}\quad \Pi_i = \frac{n_i}{n_j}\delta_x \ell_i\left(\left(1-\ell_i\right)-\gamma_x\right)\left\{\left[\frac{1+\alpha_i\left(\dfrac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}+\beta_i\left(\dfrac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}}{1+\alpha_i\left(\dfrac{P^t}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}+\beta_i\left(\dfrac{\delta_x\,\ell_i}{\beta_i}\right)^{\frac{\rho_i}{\rho_i-1}}}\right]^{\frac{1-\rho_i}{\rho_i}}-1\right\} \tag{A9}$$

$$\Pi_j = \delta_y\left[\left(1-\ell_j\right)-\gamma_y\right]\ell_j\left\{P^t - P\left[\frac{1+\alpha_j\left(\dfrac{1}{\alpha_j}P\right)^{\frac{\rho_j}{\rho_j-1}}+\beta_j\left(\dfrac{\delta_y\,\ell_j}{\beta_j}P\right)^{\frac{\rho_j}{\rho_j-1}}}{1+\alpha_j\left(\dfrac{1}{\alpha_j}P^t\right)^{\frac{\rho_j}{\rho_j-1}}+\beta_j\left(\dfrac{\delta_y\,\ell_j}{\beta_j}P^t\right)^{\frac{\rho_j}{\rho_j-1}}}\right]^{\frac{1-\rho_j}{\rho_j}}\right\} \tag{A10}$$

Consider as an example the case in which $\rho_i > 0$ and $\rho_j < 0$. In Figure A4 we display the graph of Π_i (A9) (complete curve) and Π_j (dotted curve) (A10)) as a functions of P^t. The intersection point occurs at the autarkic price P. Observe that an appropriate value of the lump sum tax $\tau_j / P^t{}_x$ will fall in interval required by (A8).

Figure A4: Lump Sum Compensation Policy



4. Compensation policy (subsidy to education)

Consider the case in which s_i shifts to ① (Figure 8) and s_j shifts to ① (Figure 10). Is it possible that a policy of subsidies to education to the individuals-i makes them increase their level of education as much as to be willing to work in the sector-Y and induce to CS? Is it possible that this kind of policy is Pareto Improving? The budget of the government will be $z_i s^t{}_i n_i = \tau_j n_j$. In the following sections we identify the conditions under which this kind of policy is effective.

4.1 The effect of the policy on the level of education

According to our computations, stimulated by the subsidy, the individuals-i will increase their level of education. If their new level of education surpasses the critical value s^t_e defined by (16), then CS will occur. We show that for appropriate values of the parameters relation (A11) holds and the policy of subsidies succeeds in producing a CS.

$$s^t{}_i > s^t{}_e \quad \text{and} \quad s^t{}_j > s^t{}_e \tag{A11}$$

To show that the above can hold we compare the level of education of the individual-i after trade liberalization, $s_i^t\big|_{\tau,z\neq 0}$ and under the policy of subsidies with the level of education under autarky, $s_i\big|_{\tau,z=0}$.

$$s_i^t\Big|_{\tau,z\neq 0} - s_i\Big|_{\tau,z=0} = \frac{\left\{\beta_i\left[\frac{1}{\beta_i}(\delta_x\,\ell_i)\right]^{\frac{\rho_i}{\rho_i-1}} + 1 + \alpha_i\left(\frac{1}{\alpha_i}P\right)^{\frac{\rho_i}{\rho_i-1}}\right\}^{-1}}{\beta_i\left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i + \frac{z_i}{p_x^t}\right)\right]^{\frac{\rho_i}{\rho_i-1}} + 1 + \alpha_i\left(\frac{1}{\alpha_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}}} N\left(P^t\right) \tag{A12}$$

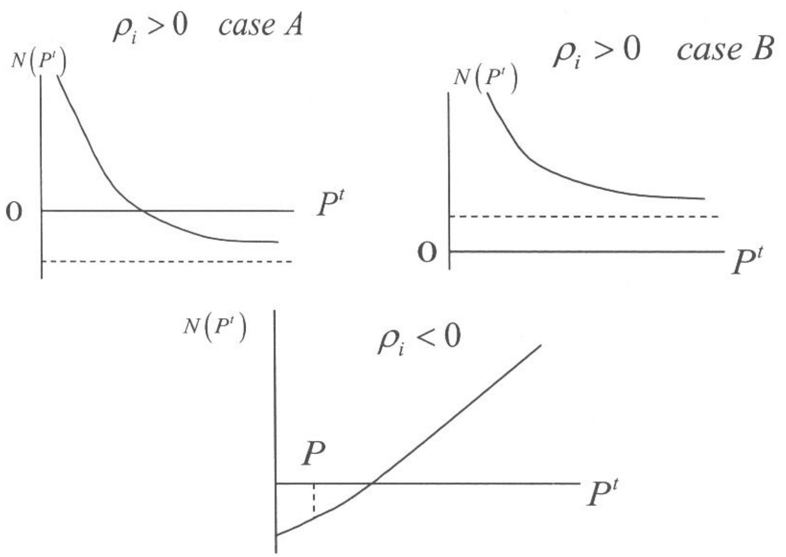
where $N\,(P^t)$ is defined as follows:

$$N\left(P^t\right)=\left[\left(1-\ell_i\right)-\gamma_x\right]\beta_i\left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i\right)\right]^{\frac{\rho_i}{\rho_i-1}}\left[1+\alpha_i\left(\frac{1}{\alpha_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}}\right]+\beta_i\left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i+\frac{z_i}{p_x^t}\right)\right]^{\frac{\rho_i}{\rho_i-1}}\Omega(P)$$

$$\Omega(P)=\frac{\gamma_x\delta_x\ell_i\left\{\beta_i\left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i\right)\right]^{\frac{\rho_i}{\rho_i-1}}+1+\alpha_i\left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}\right\}}{\delta_x\,\ell_i+\frac{z_i}{p_x^t}}-\left\{\left(1-\ell_i\right)\left[1+\alpha_i\left(\frac{P}{\alpha_i}\right)^{\frac{\rho_i}{\rho_i-1}}\right]+\gamma_x\beta_i\left[\frac{1}{\beta_i}\left(\delta_x\,\ell_i\right)\right]^{\frac{\rho_i}{\rho_i-1}}\right\}$$

We depict function $N(P^t)$ in Figure A5. When $\rho_i>0$ we have 2 cases. Case A:z_i is low: $s_x^t\big|_{\tau,z\neq0}>s_x\big|_{\tau,z=0}$ if P^t is relatively low. Case B:z_i is high: $s_x^t\big|_{\tau,z\neq0}>s_x\big|_{\tau,z=0}$ $\forall\ P^t$. In the case $\rho_i<0$, $s_x^t\big|_{\tau,z\neq0}>s_x\big|_{\tau,z=0}$ if P^t is high.

Figure A5: The level of education of individuals-i

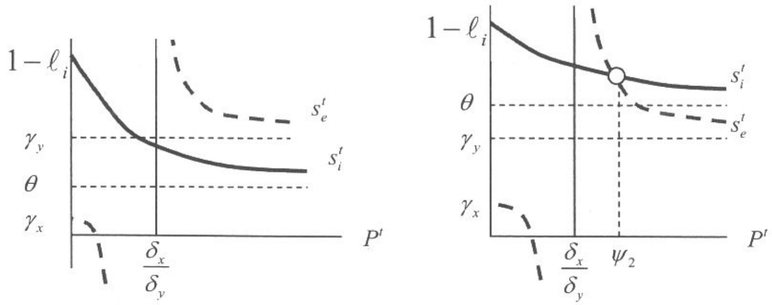


4.2. Condition for the policy to induce to complete specialization

We depict the graphs of s^t_i (complete curve) and s^t_e (dotted curve) in Figure A6 in the case $\rho_i > 0$ and in Figure A7 in the case $\rho_i < 0$.

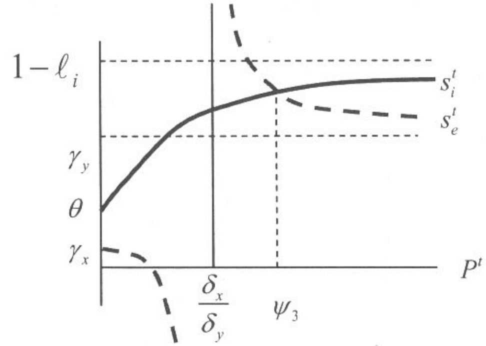
5.2.1. The case $\rho_i > 0$. We may have two situation depending on the value of $\theta = \lim_{P^t \to \infty} s^t_i (P^t) = \text{constant} > \gamma_x$. As $\frac{\delta_x}{\delta_y} < P^t$, then if $\theta < \gamma_y$ $\Rightarrow s^t_i < s^t_e$ in the interval of existence. The policy induces to CS in the case of Figure A5 (right). When $\theta > \gamma_y \Rightarrow s^t_i > s^t_e \Leftrightarrow P^t > \Psi_2$.

Figure A6: The graphs of s^t_i and s^t_e when $\rho_i > 0$



4.2.2. The case $\rho_i < 0$

Figure A7: The graphs of s^t_i and s^t_e when $\rho_i < 0$



$$s^t_i > s^t_e \Leftrightarrow \quad P^t > \psi_3 \tag{A13}$$

It can be shown that if the individual-i shifts to the production of sector good-Y, $s^t_{i/y} > s^t_{i/x}$ holds if $\rho_i > 0$. If $\rho_i < 0$, $s^t_{i/y} > s^t_{i/x}$ as long as P^t $\frac{\delta_y}{\delta_x}$ is not too high.

Next we show that a level of P^t that falls in the intervals (A13) exists. First, we compute the level of P^t. Assume the FC specialized in the production of good-X and there is no governmental intervention. The equilibrium value of P^t can be obtained computing (A1). The equilibrium condition in the market of good-Y will become

$$A\,(P^t) + T\,(P^t) = A^*\,(P^t) \tag{A14}$$

Where

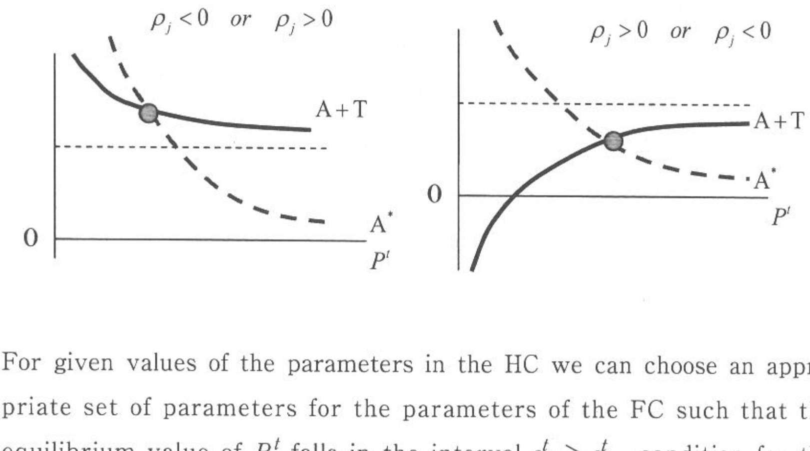
$$\mathrm{T}\,(P^t) = \frac{\tau_j}{p^t_x}\, nj\,(\mathrm{T}_0 - \mathrm{T}_1) \tag{A15}$$

$$T_0(P^t)=\frac{1}{1+\alpha_j\left(\frac{1}{\alpha_j}P^t\right)^{\frac{\rho_j}{\rho_j-1}}}\left\{\frac{\frac{1}{P^t}\beta_j\left[\frac{1}{\beta_j}\left(P^t\delta_y\ell_j\right)\right]^{\frac{\rho_j}{\rho_j-1}}}{\beta_j\left[\frac{1}{\beta_j}\left(P^t\delta_y\ell_j\right)\right]^{\frac{\rho_j}{\rho_j-1}}+1+\alpha_j\left(\frac{1}{\alpha_j}P^t\right)^{\frac{\rho_j}{\rho_j-1}}}+\left(\frac{1}{\alpha_j}P^t\right)^{\frac{1}{\rho_j-1}}\right\}$$

$$T_1(P^t)=\frac{1}{1+\alpha_i\left(\frac{1}{\alpha_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}}}\left\{\frac{\frac{\delta_y\ell_i\gamma_y}{s^t_{i/y}}\left[\frac{1}{\beta_i}\left(P^t\delta_y\ell_i+\frac{1}{s^t_{i/y}}\frac{\tau_j}{p^t_x}\frac{n_j}{n_i}\right)\right]^{\frac{1}{\rho_i-1}}}{\beta_i\left[\frac{1}{\beta_i}\left(P^t\delta_y\ell_i+\frac{1}{s^t_{i/y}}\frac{\tau_j}{p^t_x}\frac{n_j}{n_i}\right)\right]^{\frac{\rho_i}{\rho_i-1}}+1+\alpha_i\left(\frac{1}{\alpha_i}P^t\right)^{\frac{\rho_i}{\rho_i-1}}}+\left(\frac{1}{\alpha_i}P^t\right)^{\frac{1}{\rho_i-1}}\right\}$$

Consider the case $\rho_i<0$ and $\rho_j>0$ (Figure A8 left), and the case $\rho_i>0$ and $\rho_j<0$ (see Figure A8 right). The equilibrium value of P^t will occur at the intersection of A + T (complete curve) and A* (the dotted curve).

Figure A8: The equilibrium of P^t when the HC applies the subsidy policy



For given values of the parameters in the HC we can choose an appropriate set of parameters for the parameters of the FC such that the equilibrium value of P^t falls in the interval $s^t_i > s^t_e$, condition for the policy to induce the HC to specialize.

5.2.3. Utility of the individuals

Under certain conditions, the policy of subsidies will allow the trade liberalization to become Pareto-improving. We compare the level of utility of the individuals under autarky and under unrestricted trade. The necessary and sufficient condition for a Pareto improving policy is

$$\frac{W_i^t}{W_i} \geq 1 \;\;\&\;\; \frac{W_j^t}{W_j} \geq 1 \Leftrightarrow \tilde{\Pi}_i \leq \frac{\tau_j}{p_x^t} \leq \Pi_j \tag{A16}$$

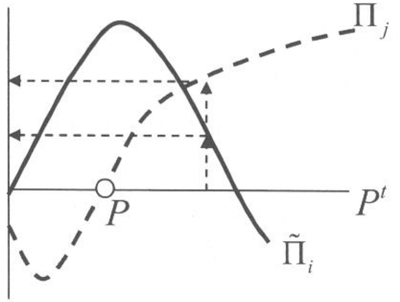
The mathematical expression for Π_j is the same as (A10). $\tilde{\Pi}_i$ is defined from:

$$\tilde{\Pi}_i = \frac{n_i}{n_j}\left[\delta_x\left(s_{i/x} - \gamma_x\right)\ell_i\Gamma_i - P^t\delta_y\left(s_{i/y}^t - \gamma_y\right)\ell_i\right] \tag{A17}$$

The mathematical expression of $s_{i/x}$ is taken from (5) setting $\tau_i = z_i = 0$ and $s^t_{i/y}$ is similar to (8) evaluating the function at P^t and $\tau_i = 0$ with $z_i = 0$.

Figure A9 displays the graph of $\tilde{\Pi}_i$ (equation (A17)) and (complete curve) and Π_j (dotted curve) (equation (A10)) as a function of P^t in the case $\rho_i > 0$ and $\rho_j < 0$. Function Π_j becomes zero at $P = P^t$. Observe from the graph that, as long as P^t is high enough, for an appropriate value of τ_j/p^t_x will fall in the interval (A16).

Figure A9: A subsidy to education policy (the case $\rho_i > 0$ and $\rho_j < 0$)



6. The Real wage rate Index

6.1. The Case of Partial Specialization (PS)

Using equations (2), and after some mathematical manipulations it is easy to obtain an expression of the nominal wage rates under free trade $w^t{}_{i/x}$ and $w^t{}_{j/y}$ as a function of the autarkic wage rates $w_{i/x}$ and $w_{j/y}$. The final formulas are as follows.

$$w^t_{i/x} = w_{i/x}\left(1 + q_i \frac{dP}{P}\eta_i\right) \quad \text{and} \quad w^t_{j/y} = w_{j/y}\left[1 + \frac{dP}{P}\left(1 + q_j \eta_j\right)\right] \quad \text{(A18)}$$

$$\text{where } q_i = \frac{s_i}{s_i - \gamma_x} \qquad q_j = \frac{s_j}{s_j - \gamma_y} \quad \text{(A19)}$$

$$\eta_i = \frac{ds_{i/x}}{dP}\frac{P}{s_{i/x}} \qquad \eta_j = \frac{ds_{j/y}}{dP}\frac{P}{s_{j/y}}$$

$q_i > 1$ and $q_j > 1$ are indicators of how close are the initial position of the level of education to the minimum technological requirements γ_x and γ_y. Symbols η_i and η_j represent the elasticity of education with respect to the relative price. It is a common practice to define elasticities as positive numbers, for what expressions are usually transformed into its absolute value. However, in order to simplify notation we keep the original expression and take the elasticities η_i and η_j without changing to absolute values. From (10), $\eta_i > 0 \Leftrightarrow \rho_i < 0$ and $\eta_j > 0 \Leftrightarrow \rho_j > 0$. After some computations we find that the nominal wage index defined in (A20) can be written as follows.

$$\frac{\mathrm{I}^t}{\mathrm{I}} = \frac{1 + \dfrac{dP}{P}\left(1 + q_j \eta_j\right)}{1 + \dfrac{dP}{P} q_i \eta_i} \tag{A20}$$

Notice that as $w^t{}_{i/x}$ (A18) represents the wage rate and is always a positive number, $1 + \dfrac{dP}{P} q_i \eta_i > 0$, too. Then, from (A20) it is not difficult to obtain the following relation:

$$\mathrm{I}^t > \mathrm{I} \Leftrightarrow q_i \eta_i - q_j \eta_j < 1 \tag{A21}$$

According to (A21), $\mathrm{I}^t > \mathrm{I}$ will hold when $\eta_i < 0$ $(0 < \rho_i < 1)$ and $\eta_j > 0$ $(0 < \rho_j < 1)$. In this case s_i shifts to positions ② of Figure 8 (or ⑥ of Figure 9) and s_j to position ① of Figure 10.

On the other hand, as $q_i, q_j > 1$, according to (A21), $\mathrm{I}^t < \mathrm{I}$ may hold will when $\eta_i > 0$ and $\eta_j < 0$, with η_j and/or $|\eta_j|$ high enough, then may hold. In this case s_i increases and shifts to positions ① of Figure 8 (or ④ or Figure 9), and s_j falls to position ② of Figure 10. In this case, both parameters ρ_i, $\rho_j < 0$.

6.2. The Case of Complete Specialization (CS)

When CS occurs, the formula for $w^t_{j/y}$, the wage rate under unrestricted trade is the same as the one given by (A18), but the formula for $w^t_{y/i}$ is different because the individuals-i will shift to the production of good-Y. The formula will be as follows.

$$w^t_{y/i} = w_{i/x} P \frac{\delta_y}{\delta_x}\left(1+\frac{dP}{P}\right)\left(\frac{s_i-\gamma_y+ds_{i/y}}{s_i-\gamma_x}\right) \tag{A22}$$

Using the above expression, we can recalculate the nominal wage rate equation (20). After some computations we get the following formula.

$$\frac{\mathrm{I}^t}{\mathrm{I}} = \frac{\delta_x}{\delta_y}\frac{1}{P}\Theta_1 \text{ and } \Theta_1 = \frac{1}{\left(1+\frac{\Delta P}{P}\right)} \frac{1+\frac{\Delta P}{P}+\frac{\Delta s_{j/y}}{s_{j/y}-\gamma_y}}{\frac{s_{i/x}-\gamma_y+\Delta s_{i/y}}{s_{i/x}-\gamma_x}} \tag{A23}$$

Here notice that as stated in (7), $\frac{\delta_x}{\delta_y}\frac{1}{P}<1$. Moreover, after some mathematical computations we find that

$$\Theta_1 < 1 \Leftrightarrow \eta_j < \left(\eta_i - \frac{\gamma_y-\gamma_x}{s_{i/y}}\frac{P}{\Delta P}\right)\frac{q_i}{q_j}\left(1+\frac{\Delta P}{P}\right) \tag{A24}$$

Where η_i, η_j, q_i, q_j are defined as in (A19).

$\mathrm{I}^t > \mathrm{I}$ will hold when $\Theta_1 > 1$ and is high enough to compensate for $\frac{\delta_x}{\delta_y}\frac{1}{P}<1$. This may will happen if $\eta_i<0$ and $\eta_j>0$. This means that $\rho_i>0$ and $\rho_j>0$. While s_i is falling to positions ② of Figure 8 or to positions ⑤ or ⑥ of Figure 9, s_j increases to position ① of Figure 10.

$\mathrm{I}^t < \mathrm{I}$ will occur when $\Theta_1 < 1$. This in turn will hold when $\eta_i > 0$ and $\eta_j < 0$ with both η_i and $|\eta_j|$ high enough. This means that $\rho_i < 0$ and $\rho_j < 0$ what indicates that s_i is experiments important increases and shifts to position ③ of Figure 8, or to ④ of Figure 9). Besides, s_j falls to position ② of Figure 10).

6.3. The price indices

Using equations (5) (and the equivalents for individual-j) the above can be expressed as a function of the relative price and the terms of trade as follows

$$PI_i = \frac{1 + r_i P^t}{1 + r_i P} \qquad \text{and} \qquad PI_j = \frac{1 + r_j P^t}{1 + r_j P} \tag{A25}$$

$$\text{where } r_i = \left(\frac{P}{\alpha_i}\right)^{\frac{1}{\rho_i - 1}} \qquad \text{and} \qquad r_j = \left(\frac{P}{\alpha_j}\right)^{\frac{1}{\rho_j - 1}} \tag{A26}$$

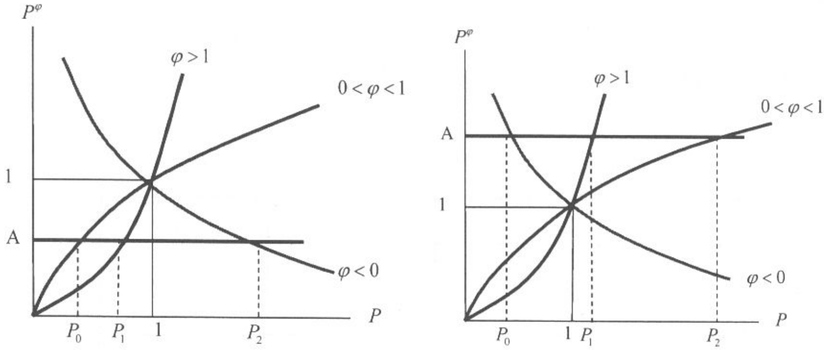
Putting together (A25) and (A26) it is easy to show that $PI_i/PI_j > 1 \Leftrightarrow r_i > r_j$. After some computations it can be seen that

$$PI_i / PI_j > 1 \Leftrightarrow r_i > r_j \Leftrightarrow P^{\varphi} > \mathrm{A} \tag{A27}$$

$$\text{where } \varphi = \frac{\rho_j - \rho_i}{(\rho_i - 1)(\rho_j - 1)} \quad \text{and} \quad \mathrm{A} = \frac{(\alpha_i)^{\frac{1}{\rho_i - 1}}}{(\alpha_j)^{\frac{1}{\rho_j - 1}}} \tag{A28}$$

Consider the two cases in which the two parameters ρ_i and ρ_j have the same sign. Case 1) $0 < \rho_i, \rho_j < 1$ and Case 2) $\rho_i, \rho_j < 0$. In both cases $\varphi > 0 \Leftrightarrow \rho_j > \rho_i$. In the following graph we depict curve P^{φ}.

Figure A10: the graph of function P^{φ}



PI_i/PI_j will be higher or lower than 1 depending on the level of the relative price under autarky P. Besides, from (A32) we have that $\varphi < 1$ $\Leftrightarrow \rho_j < \dfrac{1}{2-\rho_i}$.

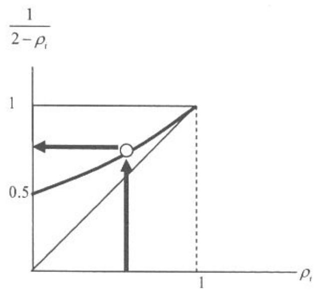
Therefore,

When $\rho_i, \rho_j < 0 \Rightarrow 0 < \varphi < 1 \Leftrightarrow \rho_i < \rho_j < 0$ and $\varphi < 0 \Leftrightarrow \rho_j < \rho_i < 0$

When $\rho_i, \rho_j > 0 \Rightarrow \varphi < 0 \Leftrightarrow 0 < \rho_j < \rho_i$ and $\varphi > 1 \Leftrightarrow \rho_j > \dfrac{1}{2-\rho_i}$

We represent the graph of function $1/(2-\rho_i)$ below.

Figure A11: the graph of function $1/(2-\rho_i)$



Consider the two relevant cases considered in Table 3 of the main text: first ρ_i, $\rho_j > 0$ with $PI_i/PI_j \geq 1$ and second ρ_i, $\rho_j < 0$ with $PI_i/PI_j < 1$. Observing the above two graphs we can get the respective conditions which are summarized in the following Table.

Conditions for $PI_i/PI_j > 1$ in the case ρ_i, $\rho_j > 0$

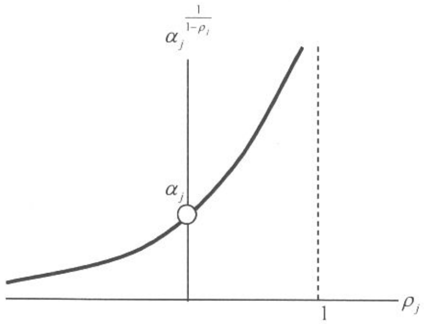
	for A <1	for A >1
$\varphi < 0 \Leftrightarrow 0 < \rho_j < \rho_i$	$P < P_2$	$P < P_0$
$\varphi > 1 \Leftrightarrow \rho_j > \dfrac{1}{2-\rho_i}$	$P > P_1$	$P > P_1$

Conditions for $PI_i/PI_j < 1$ in the case ρ_i, $\rho_j < 0$

	for A <1	for A >1
$\varphi < 0 \Leftrightarrow \rho_j < \rho_i < 0$	$P > P_2$	$P > P_0$
$0 < \varphi < 1 \Leftrightarrow \rho_i < \rho_j < 0$	$P < P_1$	$P < P_1$

Finally consider the quotient A of (A32). The graph of function $f\left(\rho_j\right) = \alpha_j^{\frac{1}{1-\rho_j}}$ (numerator of quotient A) is depicted below.

Figure A12: the graph of function $f(\rho_j)$



Then the quotient A will be higher for a higher a_j relative to a_i and for a higher ρ_j relative to ρ_i.

As an illustrative case consider the case in which $a_i = a_j$. Then we will have the following simple conditions.

Conditions for $PI_i/PI_j > 1$ when $a_i = a_j$

$0 < \rho_j < \rho_i$	$P < P_2$
$\rho_j > \frac{1}{2-\rho_i}$	$P > P_1$

Conditions for $PI_i/PI_j < 1$ when $a_i = a_j$

$\rho_j < \rho_i < 0$	$P > P_2$
$\rho_i < \rho_j < 0$	$P < P_1$